

## 71. $D_s^+$ Branching Fractions

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Figure 71.1 shows a partial breakdown of the  $D_s^+$  branching fractions. The rest of this note is about how the figure was constructed. The values shown make heavy use of CLEO measurements of inclusive branching fractions [1]. For references to other data cited in the following, see the Listings. An addendum updates branching fractions to two-body final states reported by the BESIII Collaboration in 2020 [2], and the conclusion stresses modes still to be identified.

### 71.1 Modes with leptons

The bottom  $(18.0 \pm 1.0)\%$  of Fig. 71.1 shows the fractions for the modes that include leptons. The measured  $K^0 e^+ \nu_e$  and  $K^{*0} e^+ \nu_e$  fractions have been doubled to take account of the corresponding  $\mu^+ \nu_\mu$  fractions. The sum of the exclusive  $X e^+ \nu_e$  fractions is  $(6.0 \pm 0.3)\%$ , consistent with an inclusive semileptonic measurement of  $(6.5 \pm 0.4)\%$ . There seems to be little missing here.

### 71.2 Inclusive hadronic $K\bar{K}$ fractions

The Cabibbo-favored  $c \rightarrow s$  decay in  $D_s^+$  decay produces a final state with both an  $s$  and an  $\bar{s}$ ; and thus modes with a  $K\bar{K}$  pair or with an  $\eta$ ,  $\omega$ ,  $\eta'$ , or  $\phi$  predominate (as may already be seen in Fig. 71.1 in the semileptonic fractions). We consider the  $K\bar{K}$  modes first. A complete picture of the exclusive  $K\bar{K}$  charge modes is not yet possible, because branching fractions for many of those modes have not yet been measured. However, CLEO has measured the inclusive  $K^+$ ,  $K^-$ ,  $K_S^0$ ,  $K^+ K^-$ ,  $K^+ K_S^0$ ,  $K^- K_S^0$ , and  $2K_S^0$  fractions (these include modes with leptons) [1]. And each of these inclusive fractions with a  $K_S^0$  is equal to the corresponding fraction with a  $K_L^0$ :  $f(K^+ K_L^0) = f(K^+ K_S^0)$ ,  $f(2K_L^0) = f(2K_S^0)$ , etc. Therefore, of all-inclusive fractions pairing a  $K^+$ ,  $K_S^0$ , or  $K_L^0$  with a  $K^-$ ,  $K_S^0$ , or  $K_L^0$ , we know all but  $f(K_S^0 K_L^0)$ .

We can get that fraction. The total  $K_S^0$  fraction is

$$f(K_S^0) = f(K^+ K_S^0) + f(K^- K_S^0) + 2f(2K_S^0) + f(K_S^0 K_L^0) + f(\text{single } K_S^0),$$

where  $f(\text{single } K_S^0)$  is the sum of the branching fractions for modes such as  $K_S^0 \pi^+ 2\pi^0$  with a  $K_S^0$  and no second  $K$ . The  $K_S^0 \pi^+ 2\pi^0$  mode is in fact the only unmeasured single- $K_S^0$  mode (throughout, we shall assume that fractions for modes with a  $K$  or  $K\bar{K}$  and more than three pions are negligible), and we shall take its fraction to be the same as for the  $K_S^0 2\pi^+ \pi^-$  mode,  $(0.30 \pm 0.11)\%$ . Any reasonable deviation from this value would be too small to matter much in the following. Adding the several small single- $K_S^0$  branching fractions, including those from semileptonic modes, we get  $f(\text{single } K_S^0) = (1.7 \pm 0.2)\%$ . Using this, we have:

$$\begin{aligned} f(K_S^0 K_L^0) &= f(K_S^0) - f(K^+ K_S^0) - f(K^- K_S^0) - 2f(2K_S^0) - f(\text{single } K_S^0) \\ &= (19.0 \pm 1.1) - (5.8 \pm 0.5) - (1.9 \pm 0.4) - 2 \times (1.7 \pm 0.3) - (1.7 \pm 0.2) \\ &= (6.2 \pm 1.4)\% . \end{aligned}$$

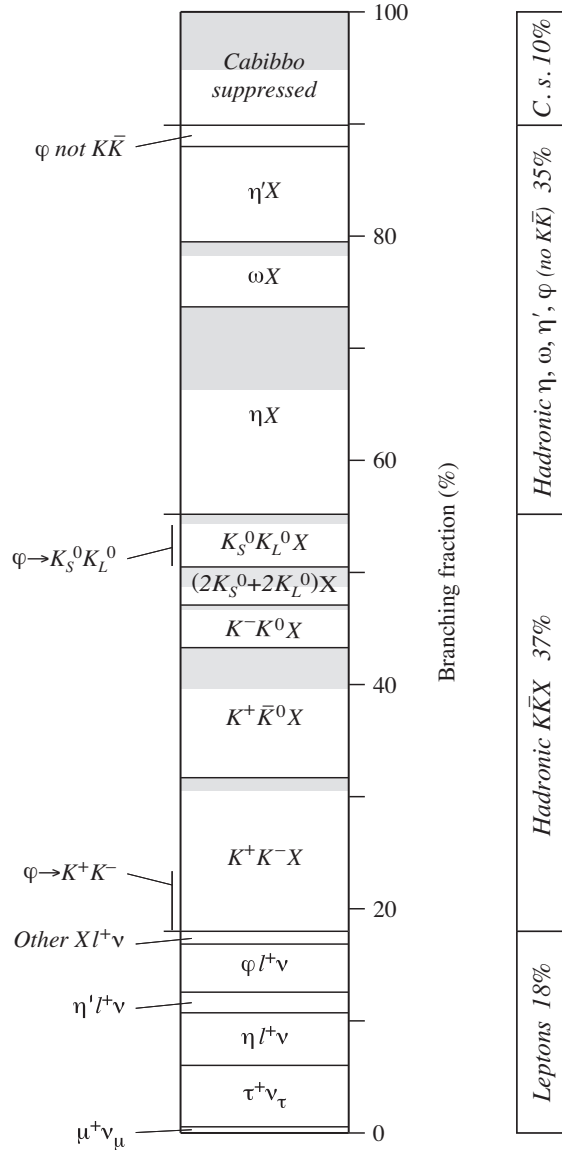
Here and below we treat the errors as uncorrelated, although often they are not. However, our main aim is to get numbers for Fig. 71.1; errors are secondary.

There is a check on our result: The  $\phi$  inclusive branching fraction is  $(15.7 \pm 1.0)\%$ , of which 34%, or  $(5.34 \pm 0.34)\%$  of  $D_s^+$  decays, produces a  $K_S^0 K_L^0$ . Our  $f(K_S^0 K_L^0) = (6.2 \pm 1.4)\%$  has to be at least this large—and it is, within the sizable error.

We now have all the inclusive  $K\bar{K}$  fractions. We use  $f(K^+ \bar{K}^0) = 2 f(K^+ K_S^0)$ , and likewise for  $f(K^- \bar{K}^0)$ . For  $K^+ K^-$  and  $K_S^0 K_L^0$ , we subtract off the contributions from  $\phi \ell^+ \nu$  decay to get the

purely hadronic  $K\bar{K}$  inclusive fractions:

$$\begin{aligned}
 f(K^+K^-, \text{hadronic}) &= (15.8 \pm 0.7) - (2.1 \pm 0.3) = (13.7 \pm 0.8)\% \\
 f(K^+\bar{K}^0, \text{hadronic}) &= (11.6 \pm 1.0)\% \\
 f(K^-K^0, \text{hadronic}) &= (3.8 \pm 0.8)\% \\
 f(2K_S^0 + 2K_L^0, \text{hadronic}) &= (3.4 \pm 0.6)\% \\
 f(K_S^0K_L^0, \text{hadronic}) &= (6.2 \pm 1.4) - (1.5 \pm 0.2) = (4.7 \pm 1.4)\% .
 \end{aligned}$$



**Figure 71.1:** A partial breakdown of  $D_s^+$  branching fractions. The hadronic bins in the left column show inclusive fractions. Shading within a bin shows how much of the inclusive fraction is not yet accounted for by adding up all the relevant exclusive fractions. The inclusive hadronic  $\phi$  fraction is spread over three bins, in proportion to its decay fractions into  $K^+K^-$ ,  $K_S^0K_L^0$ , and no- $K\bar{K}$  modes.

The fractions are shown in Fig. 71.1. They total  $(37.2 \pm 2.2)\%$  of  $D_s^+$  decays.

We can add more information to the figure by summing up measured branching fractions for exclusive modes within each bin:

$K^+K^-$  *modes*—The sum of measured  $K^+K^-\pi^+$ ,  $K^+K^-\pi^+\pi^0$ , and  $K^+K^-\pi^+\pi^-\pi^0$  branching fractions is  $(12.6 \pm 0.6)\%$ . That leaves  $(1.1 \pm 1.0)\%$  for the  $K^+K^-\pi^+\pi^0$  mode, which is the only other  $K^+K^-$  mode with three or fewer pions. In Fig. 71.1, this unmeasured part of the  $K^+K^-$  bin is shaded.

$K^+\bar{K}^0$  *modes*—Two times the sum of the measured  $K^+K_S^0$ ,  $K^+K_S^0\pi^0$ , and  $K^+K_S^0\pi^+\pi^0$  branching fractions is  $(8.0 \pm 0.5)\%$ . This leaves  $(3.6 \pm 1.1)\%$  for the unmeasured  $K^+\bar{K}^0$  modes (there are three such modes with three or fewer pions). This is shaded in the figure.

$K^-K^0$  *modes*—Twice the  $K^-K_S^0\pi^0$  fraction is  $(3.4 \pm 0.2)\%$ , which leaves about  $(0.4 \pm 0.8)\%$  for  $K^-K^0\pi^+\pi^0$ , the only other  $K^-K^0$  mode with three or fewer pions.

$2K_S^0 + 2K_L^0$  *modes*—The  $2K_S^0\pi^+$  and  $2K_S^0\pi^+\pi^0$  fractions sum to  $(0.86 \pm 0.07)\%$ ; this times two (for the corresponding  $2K_L^0$  modes) is  $(1.72 \pm 0.14)\%$ . This leaves about  $(1.7 \pm 0.7)\%$  for other  $2K_S^0 + 2K_L^0$  modes.

$K_S^0K_L^0$  *modes*—Most of the  $K_S^0K_L^0$  fraction is accounted for by  $\phi$  decays (see below).

### 71.3 Inclusive hadronic $\eta$ , $\omega$ , $\eta'$ , and $\phi$ fractions

These are easier. We start with the inclusive branching fractions, and then, to avoid double counting, subtract: (1) fractions for modes with leptons; (2)  $\eta$  mesons that are included in the inclusive  $\eta'$  fraction; and (3)  $K^+K^-$  and  $K_S^0K_L^0$  from  $\phi$  decays:

$$f(\eta \text{ hadronic}) = f(\eta \text{ inclusive}) - 0.65 f(\eta' \text{ inclusive}) - f(\eta\ell^+\nu) = (18.5 \pm 3.0)\%$$

$$f(\omega \text{ hadronic}) = f(\omega \text{ inclusive}) - 0.026 f(\eta' \text{ inclusive}) = (5.8 \pm 1.4)\%$$

$$f(\eta' \text{ hadronic}) = f(\eta' \text{ inclusive}) - f(\eta'\ell^+\nu) = (8.5 \pm 1.5)\%$$

$$f(\phi \text{ hadronic, } \not\rightarrow K\bar{K}) = 0.17 [f(\phi \text{ inclusive}) - f(\phi\ell^+\nu)] = (1.9 \pm 0.2)\% .$$

The factors 0.65, 0.026, and 0.17 are the  $\eta' \rightarrow \eta$ ,  $\eta' \rightarrow \omega$ , and  $\phi \rightarrow K\bar{K}$  branching fractions. Figure 71.1 shows the results; the sum is  $(34.7 \pm 3.6)\%$ , which is about equal to the hadronic  $K\bar{K}$  total.

Note that the bin marked  $\phi$  near the top of Fig. 71.1 includes neither the  $\phi\ell^+\nu$  decays nor the 83% of other  $\phi$  decays that produce a  $K\bar{K}$  pair. There is twice as much  $\phi$  in the  $K_S^0K_L^0$  bin, and nearly three times as much in the  $K^+K^-$  bin. These contributions are indicated in those bins.

Again, we can show how much of each bin is accounted for by measured exclusive branching fractions:  $\eta$  *modes*—The sum of  $\eta\pi^+$ ,  $\eta\pi^+\pi^0$  (nearly all  $\eta\rho^+$ ), and  $\eta K^+$  branching fractions is  $(11.1 \pm 1.2)\%$ , which leaves a good part of the inclusive hadronic  $\eta$  fraction,  $(18.5 \pm 3.0)\%$ , to be accounted for. This is shaded in the figure.  $\omega$  *modes*—The sum of  $\omega\pi^+$ ,  $\omega\pi^+\pi^0$ , and  $\omega 2\pi^+\pi^-$  fractions is  $(4.6 \pm 0.9)\%$ , which is nearly as large as the inclusive hadronic  $\omega$  fraction,  $(5.8 \pm 1.4)\%$ .

$\eta'$  *modes*—The sum of  $\eta'\pi^+$ ,  $\eta'\rho^+$ , and  $\eta'K^+$  fractions is  $(9.9 \pm 1.5)\%$ , which is larger than but not in serious disagreement with the inclusive hadronic  $\eta'$  fraction,  $(8.5 \pm 1.5)\%$ .

### 71.4 Cabibbo-suppressed modes

The sum of the fractions for modes with a  $K\bar{K}$ ,  $\eta$ ,  $\omega$ ,  $\eta'$ , or leptons is  $(89.9 \pm 4.4)\%$ . The remaining  $(10.1 \pm 4.4)\%$  is to Cabibbo-suppressed modes, mainly single- $K$ + pions and multiple-pion modes (see below). However, it should be noted that some small parts of the modes already discussed are Cabibbo-suppressed. For example, the  $(1.1 \pm 0.2)\%$  of  $D_s^+$  decays to  $K^0\ell\nu$  or  $K^{*0}\ell\nu$

is already in the  $X\ell\nu$  bin in Fig. 71.1. And the inclusive measurements of  $\eta$ ,  $\omega$ , and  $\eta'$  fractions do not distinguish between (and therefore include both) Cabibbo-allowed and -suppressed modes. We shall not try to make a separation here.

$K^0 + \text{pions}$ —Above, we found that  $f(\text{single } K_S^0) = (1.7 \pm 0.2)\%$ . Subtracting semileptonic fractions with a  $K_S^0$  leaves  $(1.3 \pm 0.2)\%$ . The hadronic single- $K^0$  fraction is twice this, about  $(2.6 \pm 0.4)\%$ . The sum of measured  $K^0\pi^+$ ,  $K^0\pi^+\pi^0$ , and  $K^02\pi^+\pi^-$  fractions is  $(1.8 \pm 0.3)\%$ , about two-thirds as much.

$K^+ + \text{pions}$ —The  $K^+\pi^0$  and  $K^+\pi^+\pi^-$  fractions sum to  $(0.72 \pm 0.05)\%$ . The total  $K^+$  fraction wanted here is probably in the 1-to-2% range.

*Multi-pions*—The  $2\pi^+\pi^-$ ,  $\pi^+2\pi^0$ , and  $3\pi^+2\pi^-$  fractions total  $(2.5 \pm 0.2)\%$ . Modes not measured might double this.

The sum of the actually measured fractions is, including the semileptonic,  $(4.9 \pm 0.3)\%$ . The error on our Cabibbo-suppressed total,  $(10.1 \pm 4.4)\%$  is too large to know how much we might be missing.

### 71.5 Addendum: Improved precision in some measurements

The Table compares measurements of branching fractions to pairs of two pseudoscalar mesons used in our 2019 update of the 2018 PDG review with values available in 2021 [2]. They have not changed much but are much better known.

Final state	2019 PDG avg or fit (%)	New BESIII entry in 2021 PDG (%)	2021 PDG avg or fit (%)
$K^+\eta'$	$0.18 \pm 0.06$	$0.268 \pm 0.025$	$0.265 \pm 0.025$
$K^+\eta$	$0.177 \pm 0.035$	$0.162 \pm 0.012$	$0.160 \pm 0.011$
$\eta\pi^+$	$1.70 \pm 0.09$	$1.741 \pm 0.063$	$1.68 \pm 0.10$
$K^+K_S^0$	$1.50 \pm 0.05$	$1.502 \pm 0.055$	$1.46 \pm 0.04$
$K_S^0\pi^+$	$0.122 \pm 0.006$	$0.1109 \pm 0.005$	$0.110 \pm 0.005$
$K^+\pi^0$	$0.063 \pm 0.021$	$0.0748 \pm 0.0057$	$0.074 \pm 0.0057$

Other recent BESIII additions include the measurements  $\mathcal{B}(D_s \rightarrow \omega\pi^+, \omega K^+) = (1.77 \pm 0.32 \pm 0.13, 0.87 \pm 0.24 \pm 0.08) \times 10^{-3}$  [3] and  $\mathcal{B}(D_s \rightarrow K_{S,L}K^+) = (1.425 \pm 0.038 \pm 0.031, 1.485 \pm 0.039 \pm 0.046)\%$  [4].

### 71.6 Concluding remarks

The shaded bands in the figure imply that about eight percent of the total  $D_s$  decay rate is into  $\eta X$ , with  $X$  still to be accounted for. Unidentified Cabibbo-suppressed modes and the modes  $K^+\bar{K}^0 X$  and  $(2K_S^0 + 2K_L^0)X$  respectively correspond to  $\sim (5, 3, 2)\%$  of the total. However, one can consider the vast majority of  $D_s$  decays to be represented by exclusive final states.

### References

- [1] S. Dobbs *et al.* (CLEO), *Phys. Rev. D* **79**, 112008 (2009), [arXiv:0904.2417].
- [2] M. Ablikim *et al.* (BESIII), *JHEP* **08**, 146 (2020), [arXiv:2005.05072].
- [3] M. Ablikim *et al.* (BESIII), *Phys. Rev. D* **99**, 9, 091101 (2019), [arXiv:1811.00392].
- [4] M. Ablikim *et al.* (BESIII), *Phys. Rev. D* **99**, 11, 112005 (2019), [arXiv:1903.04164].