# 63. Spectroscopy of Light Meson Resonances

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## 63.1 Introduction

According to the constituent quark model, a light meson consists of a color singlet quark-antiquark pair made of the $u$, $d$, or $s$ quarks and the $\overline{u}$, $\overline{d}$, or $\overline{s}$ antiquarks, grouped into a flavor multiplet of SU(3). However, additional mesons made of bound gluons (glueballs) could exist in the same mass range, suggested by the self coupling of gluons in QCD. Multiquark structures are also possible, such as quark-antiquark pairs with an excited gluon (hybrid mesons). Tetraquarks are compact color singlets of diquark-antidiquark pairs ($qq\overline{q}\overline{q}$) or 'molecular' bound states of two mesons ($q\bar{q}q\bar{q}$). More complex systems such as $qqq\overline{q}\overline{q}\overline{q}$ (baryonia) are also predicted.

Fundamentals on the constituent quark model on light and heavy mesons and baryons (including hadrons with charm and bottom quarks), and on predictions from lattice gauge theories, are described in 'Quark Model' in this issue of the *Review of Particle Physics*, henceforth called the *Review.* In the present text we describe the experimental spectrum of light mesons and their classifications within the constituent quark model, with emphasis on states exhibiting properties incompatible with $q\bar{q}$ structures. The discussion is driven by the results entered in the database of the *Review.* The spectrum of kaon excitations is much less clear cut and therefore deferred to a future edition, when further data might become available. More detailed discussions on exotic mesons – including those involving the $c$ and $b$ quarks – can be found in Refs. [1–7] and in 'Heavy Non-$q\bar{q}$ Mesons' of the *Review.* For more information on the meson (and baryon) spectrum of light (and heavy) quarks we refer to the textbook [8].

Figure 63.1 shows the mass spectrum of $q\bar{q}$ mesons. The mass (vertical axis) increases with orbital excitations $\ell$ (horizontal axis) and radial excitation $n$. In the quark model one uses the notation $nS$, $nP$, $nD$... for the radial excitations ($n = 1$ for the ground states), in contrast to the usual $1s$, $2p$, $3d$... notation in atomic physics. Each box represents an SU(3) nonet containing multiplets of the isospin $i$, three isovectors, two strange isodoublets, and two isosinglets (one SU(3) singlet and one SU(3) octet). The spin, parity $[P = -(-1)^{\ell}]$ and $C$-parity $[C = (-1)^{\ell+s}]$ take the possible values $J^{PC} = 0^{++}$ (scalar), $0^{-+}$ (pseudoscalar), $1^{--}$ (vector), $1^{+\pm}$ (axial- or pseudovector), $2^{++}$ (tensor), etc. Therefore mesons with the *exotic* quantum numbers $0^{--}$, $0^{+-}$, $1^{-+}$, $2^{+-}$, $3^{-+}$, etc. cannot be $q\bar{q}$ states. Note that the $C$-parity is defined only for the electrically neutral non-strange nonet members.

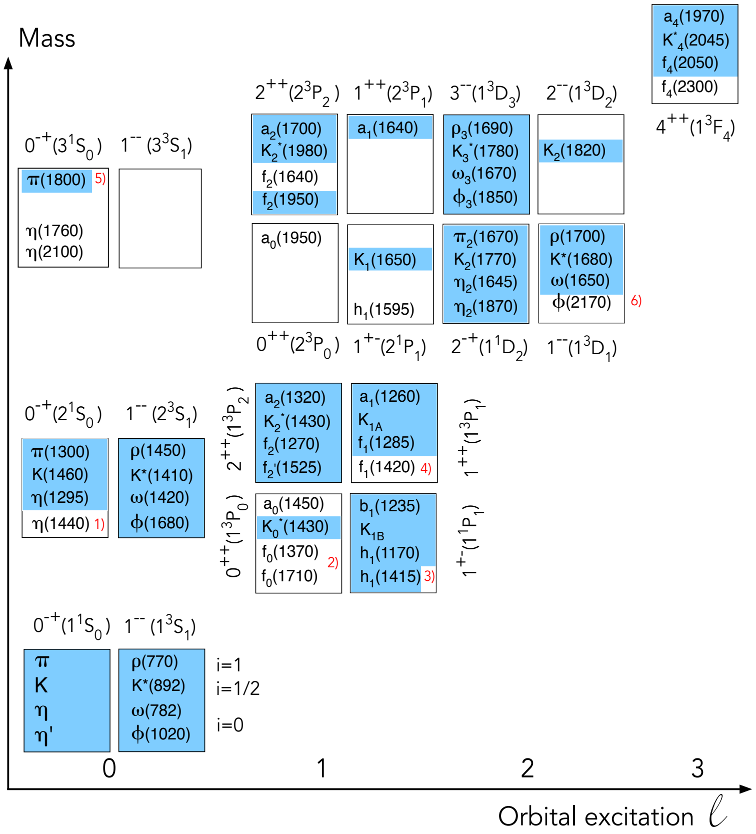

**Figure 63.1:** The mesons made of the $u$, $d$, and $s$ light quarks are organised in $J^{PC}(n^{2s+1}\ell_j)$ nonets with isospin $i$. The established mesons (which appear in the *Summary Table* of the *Review*) are shown in the dark (blue) areas. The white areas contain those omitted in the *Summary Table* but reported in the *Listings*, or the established ones with a tentative nonet classification. States with the same $J^{PC}$ mix, such as the $2^3S_1$ and $1^3D_1$ mesons. The states become broad and overlap with increasing masses, which complicates the determination of the resonance parameters *e.g.* mass, width and spin. For a complete list of mesons see the *Listings*.
[1] The $\eta(1440)$ stands for the $\eta(1405)$ and the $\eta(1475)$, section 63.4.
[2] The classification of the scalar nonet is controversial. Alternative schemes involve the $f_0(1500)$ and the light scalars below 1 GeV, section 63.2.
[3] Considered established, but more data would be desirable.
[4] An alternative to the $f_1(1420)$ is the $f_1(1510)$, section 63.6.
[5] The $\pi(1800)$ has also been proposed as a hybrid meson, section 63.7.
[6] The $\phi(2170)$ has also been proposed as a tetraquark state, section 63.8.

The two isosinglets in each nonet mix with an angle $\theta$ close to the *ideal* value of 35.3° for the $1^{--}$, $1^{+\pm}$, $2^{++}$ and $3^{--}$ nonets, in which case the isosinglets decouple to $u\bar{u} + d\bar{d}$ and $s\bar{s}$. The orbital excitations $\ell \geq 1$ consist of four nonets for each value of $n$, since $j = \ell$ for antiparallel quark spins and $j = \ell - 1, \ell$ or $\ell + 1$ for parallel spins. Since the $C$-parity is not defined for strange mesons, the $K_{1A}$ and $K_{1B}$ in the axial vector $1^{++}$ and $1^{+-}$ nonets of fig. 63.1 are mathematical constructs which mix to give the observed $K_1(1270)$ and $K_1(1400)$ mesons.

As described in more detail below (section 63.2), it is hard to accommodate all the known scalar mesons in the lower $q\bar{q}$ nonets: The light scalar mesons $a_0(980)$, $K_0^*(700)$ (also known as $\kappa$), the $f_0(500)$ (also known as $\sigma$), and the $f_0(980)$, not shown in the figure, could build the lightest nonet, but could also be two-meson resonances or tetraquarks (section 63.8). Furthermore, the ground state glueball expected below 2 GeV will mix with the $q\bar{q}$ isoscalar scalar mesons (section 63.3.1). The pseudoscalar slot labeled $\eta(1440)$ may consist of two states (section 63.4). The status of vector meson excitations is described in section 63.5. The axial-vector meson $f_1(1420)$ could be replaced in the $1^{++}$ nonet by the $f_1(1510)$ which, however, needs confirmation (see the *Listings* and section 63.6). Mesons with exotic quantum numbers are discussed in section 63.7.

## 63.2 Scalar mesons

Scalar mesons decay dominantly into pairs of pseudoscalar mesons ($\pi\pi$, $K\overline{K}$, $\pi\eta$, $\eta\eta$ or $\eta\eta'$). The widths tend to be large for those decaying into $\pi\pi$, due to the absence of angular momentum barrier and the large available phase space. Identifying broad overlapping states is not straightforward. Furthermore, the onset of the $K\overline{K}$, $\eta\eta$ or $\eta\eta'$ thresholds distorts the line shapes and produces cusps. This requires high statistics data and the use of coupled channel analyses taking into account unitarity and analyticity (see 'Resonances' in the *Review*). The SU(3) classification is also not easy, because the ground state scalar glueball and multiquark states are predicted to exist below 2 GeV.

Two isovector scalars are known, the $a_0(980)$ and the $a_0(1450)$. Five isoscalar resonances are established: the very broad $f_0(500)$, the $f_0(980)$, the broad $f_0(1370)$, and the comparatively narrow $f_0(1500)$ and $f_0(1710)$. The strange partners are the $K_0^*(700)$ and the $K_0^*(1430)$. The $f_0(500)$ and $K_0^*(700)$ deserve a separate treatment and can be found in 'Scalar Mesons below 1 GeV', which contains more details on the $a_0(980)$ and $f_0(980)$, briefly reviewed in section 63.8.

The $a_0(1450)$ was first reported by the Crystal Barrel experiment in $\bar{p}p$ annihilation with stopped antiprotons [9] (see [10] for a review of Crystal Barrel results). Its mass of about 1450 MeV is not far from that of the $a_2(1320)$ meson. An isovector scalar, possibly the $a_0(1450)$ (albeit at a lower mass of 1317 MeV) is observed by Belle in $\gamma\gamma$ collisions leading to $\eta\pi^0$ [11]. The state interferes destructively with the non-resonant background. Its $\gamma\gamma$ coupling is comparable to that of the $a_2(1320)$, in accord with simple predictions (see *e.g.* [12]). A contribution from $a_0(1450) \to K\bar{K}$ is also found in the CLEO-c analysis of $D^\pm \to K^+K^-\pi^\pm$ [13] and $D^0 \to K_S^0K^\pm\pi^\mp$ from LHCb [14].

The $f_0(1370)$ and $f_0(1500)$ were observed by Crystal Barrel [15–17] and by WA102 in central production with 450 GeV protons [18,19]. They decay mostly into $2\pi$ and $4\pi$. The $f_0(1500)$ was also observed to decay into $\eta\eta$ [20–22] and $\eta\eta'$ [23]. All data agree that the $4\pi$ decay mode represents about half of the $f_0(1500)$ decay width, but is dominant for $f_0(1370)$.

The determination of the $\pi\pi$ coupling of the very broad $f_0(1370)$ is complicated by its interference with the $f_0(500)$ and $f_0(1500)$. Its existence is questioned by COMPASS from $\pi^-p \to \pi^-\pi^-\pi^+p$ data [24]. In $\overline{B}{}_s^0 \to J/\psi(1S)\pi^+\pi^-$ from LHCb a strong scalar contribution from the $f_0(1370)$ was initially observed [25], although new data require the $f_0(500)$ and $f_0(1500)$, without any need for the $f_0(1370)$ [26]. In $\overline{B}{}^0 \to J/\psi(1S)\pi^+\pi^-$ the $\pi\pi$ mass spectrum does not show any significant scalar component above ∼1.2 GeV [27] (an analysis that is however challenged in [28]). $D^0 \to \pi^+\pi^-\pi^+\pi^-$ data from CLEO-c also require a contribution from $f_0(500)f_0(1370) \to 4\pi$ [29]. A broad $2\pi$ signal is also observed by BaBar around 1400 MeV in the decay $B^\pm \to \pi^\pm\pi^\pm\pi^\mp$ [30] which is attributed to the $f_0(1370)$, but could also be due to the $f_0(1500)$.

In Refs. [31, 32] the $f_0(1370)$ and $f_0(1710)$ (together with the $f_2(1270)$ and $f_2'(1525)$) are interpreted as bound systems of two vector mesons. This view is challenged in Ref. [33] where in a covariant formalism, e.g. the $f_2(1270)$ does not appear as a $\rho\rho$ bound state. Photoproduction data of the $f_2(1270)$ in $\gamma p \to \pi^0\pi^0 p$ from CLAS [34], as a function of momentum transfer, also disagree with predictions [35] for the $f_2(1270)$ to be a $\rho\rho$ bound state.

While the $K\bar{K}$ decay branching ratio of the $f_0(1500)$ is small [18] [36], the $f_0(1710)$ decays dominantly into $K\bar{K}$. The $f_0(1710)$ is not observed in $p\bar{p}$ annihilation at rest [36] and only weakly produced in $p\bar{p}$ annihilation in flight [37], with a rate strongly suppressed compared to that of the $f_0(1500)$ (~7%). For comparison, the rate for the $(s\bar{s})$ $f_2'(1525)$ is about 9% that of the $(u\bar{u} + d\bar{d})$ $f_2(1270)$, as expected from the OZI rule [37]. On the other hand the $f_0(1370)$ does not couple strongly to $K\bar{K}$ [18]. This suggests an $n\bar{n}$ structure $(u\bar{u} + d\bar{d})$ for the $f_0(1370)$ and $s\bar{s}$ for the $f_0(1710)$. The $f_0(1500)$ would also qualify as an $n\bar{n}$ state (see 63.3.1), although it is very narrow compared to the other states. Occam's razor principle therefore suggests that the $f_0(1370)$, $a_0(1450)$, and the $f_0(1710)$ are in the same SU(3) flavor nonet, the $f_0(1370)$ and $a_0(1450)$ being the two (mostly) $n\bar{n}$ states, and the $f_0(1710)$ the (dominantly) $s\bar{s}$ one. It is important to note that the spectrum of scalar mesons above the $f_0(1710)$ is not well established: The reported ones in the *Listings* need experimental confirmation and are not included in the *Summary Table*.

According to [38] there is mounting evidence for the existence of two scalar nonets and one glueball below 2 GeV: Table 63.1 shows a proposed classification scheme. The low mass nonet is made of four-quark states which recombine at large distances to become meson-meson resonances, while the ground state $1^3P_0$ $q\bar{q}$ nonet lies in the 1400 MeV region, see Ref. [1] for a review and Ref. [39] for the $f_0(500)$. The nature of the $f_0(1500)$ is discussed along this scheme in section 63.3.1.

Other schemes have been proposed, for example a tetraquark for the $f_0(1500)$ [40] or flavor octets for the $f_0(1500)$ and $f_0(1710)$ [3]. In Ref. [41] the $a_0(1450)$, $f_0(1370)$, $f_0(1500)$ and $K_0^*(1430)$ are radial excitations of the scalar nonet below 1 GeV. In Ref. [42] the light and heavy scalar nonets in Table 63.1 are interpreted as mixing of two tetraquark nonets. In the unitarized quark model with coupled $q\bar{q}$ and meson-meson channels, the scalars below 1 GeV are manifestations of bare $q\bar{q}$ confinement states, mass shifted from the 1.4 GeV region and distorted by the strong $^3P_0$ coupling to $S$-wave two-meson decay channels [43,44]. Thus, in these models the light scalar nonet comprising the light and heavy scalar nonets shown in Table 63.1 are manifestations of the same bare input states (see also [45]). Surprisingly for a state decaying strongly into $2\pi$ and $4\pi$, the $f_0(1370)$ is assumed to be an $s\bar{s}$ state in Ref. [46]. The $f_0(1500)$ is then a radial excitation and the glueball lies around 1700 MeV.

**Table 63.1:** Tentative classification of scalar mesons (see the text). The $1^3P_0$ ground state $q\bar{q}$ nonet is listed in the bottom half. The third isoscalar $f_0(1500)$ is discussed in section 63.3.1.

| | $\Gamma$ [MeV] | isospin $i$ | structure |
|---|---|---|---|
| $a_0(980)$ | $\sim 50$ | 1 | $K\bar{K}, qq\bar{q}\bar{q}$ |
| $f_0(980)$ | $\sim 50$ | 0 | $K\bar{K}, qq\bar{q}\bar{q}$ |
| $f_0(500)$ | $\sim 800$ | 0 | $\pi\pi, qq\bar{q}\bar{q}$ |
| $K_0^*(700)$ | $\sim 600$ | $\frac{1}{2}$ | $K\pi, qq\bar{q}\bar{q}$ |
| $a_0(1450)$ | 265 | 1 | $u\bar{d}, d\bar{u}, d\bar{d} - u\bar{u}$ |
| $f_0(1370)$ | $\sim 400$ | 0 | $d\bar{d} + u\bar{u}$ |
| $f_0(1710)$ | 125 | 0 | $s\bar{s}$ |
| $K_0^*(1430)$ | 294 | $\frac{1}{2}$ | $u\bar{s}, d\bar{s}, s\bar{u}, s\bar{d}$ |

## 63.3 Glueballs

Lattice calculations, QCD sum rules, flux tube, and constituent glue models agree that the lightest glueball has quantum numbers $J^{PC} = 0^{++}$ and the first excited state $2^{++}$. Lattice calculations predict for the ground state ($0^{++}$) a mass around 1600 − 1700 MeV [47–50], while the $2^{++}$ state lies around 2300 MeV. For more information on lattice calculations see 'Quark Model' in the *Review* and fig. 15.3 therein for an example of mass spectrum. These predictions were made in the quenched approximation, neglecting $q\bar{q}$ loops. However quenched predictions and full QCD calculations lead to small mass shifts (see 'Quark Model' and fig. 15.15 therein).

Heavier glueballs with quantum numbers $0^{-+}$, $2^{-+}$, $1^{+-}$, etc. are predicted above 2500 MeV and the lowest exotic ones (with exotic quantum numbers such as $0^{+-}$ and $2^{+-}$) are expected above 4000 MeV [50]. In holographic QCD the $0^{-+}$ is predicted to be very broad [51] and the $1^{+-}$ is at least as broad as its width [52]. Calculations of the three lowest scalar and pseudoscalar glueballs masses in pure Yang-Mills theory are in quantitative agreement with lattice results [53]. The lightest glueballs lie in the same mass region as ordinary isoscalar $q\bar{q}$ states, in the mass range of the $1^3P_0(0^{++})$, $2^3P_2(2^{++}), 3^3P_2(2^{++})$, and $1^3F_2(2^{++})$ $q\bar{q}$ states. Therefore, mixing of glueballs with nearby $q\bar{q}$ states of the same quantum numbers leads to supernumerary isoscalar state in the $q\bar{q}$ nonets (however, see Ref. [54] discussed in section 63.3.1).

Among the signatures naively expected for glueballs are (i) isoscalar states that do not fit into $q\bar{q}$ nonets, (ii) enhanced production in gluon-rich channels such as central production and radiative $J/\psi(1S)$ decay, (iii) decay branching fractions incompatible with SU(3) predictions for $q\bar{q}$ states, and (iv) reduced $\gamma\gamma$ couplings. However, mixing effects with isoscalar $q\bar{q}$ mesons [47, 55–63] and decay form factors [64] can obscure these simple signatures.

According to SU(3) the decay branching ratios for $q\bar{q}$ mesons and pure glueballs are different, and therefore useful to determine the internal structures of mesons. For pure glueballs with flavor symmetric couplings the decay ratios are $\pi\pi : K\bar{K} : \eta\eta : \eta\eta' = 3 : 4 : 1 : 0$, apart from phase space factors. The partial widths for the decay of a scalar (or a tensor) meson into a pair of pseudoscalar mesons are given in Fig. 63.2 (for a derivation see e.g. [8]). The decay of a $q\bar{q}$ meson into a pair of mesons involves the creation of a $q\bar{q}$ pair, and SU(3) symmetry assumes that the matrix elements for the creation of $s\bar{s}$, $u\bar{u}$, and $d\bar{d}$ pairs are equal. (The generalization to unequal $s\bar{s}$, $u\bar{u}$, and $d\bar{d}$ couplings is given in Ref. [55].) An excellent fit to the tensor meson decay widths is obtained with $\beta \simeq 0.5$ GeV/c, $\theta_V \simeq 26\,^\circ$ and $\theta_P \simeq -17\,^\circ$ [55].

Note that the assumption of flavor symmetric couplings may not apply in models describing the scalar glueball by a dilaton field, which lead to mass dependent couplings [65, 66] (see Ref. [67] which predicts that a scalar glueball above 1 GeV would be unobservably wide).

Another way to determine the flavor contents of neutral mesons is the decay $B \to J/\psi(1S)X$ which filters out the $d\bar{d}$ content of $X$, while $B_s^0 \to J/\psi(1S)X$ selects its $s\bar{s}$ component [68].

### 63.3.1 *Scalar glueballs*

One of the three isoscalars, the $f_0(1370)$, $f_0(1500)$ or $f_0(1710)$, appears to be supernumerary. The branching ratios in Fig. 63.2 can be used to deduce the structures of these states, assuming that they are $q\bar{q}$. The comparison of decay branching ratios from Crystal Barrel annihilation and WA102 central collision data shows that the $f_0(1500)$ is compatible with an $n\bar{n}$ structure, while the $f_0(1710)$ is mostly $s\bar{s}$ [12]. However, the close vicinity of the very broad $n\bar{n}$ $f_0(1370)$, its narrow width, enhanced production at low transverse momentum transfer in central collisions [69–71] and absence in $\gamma\gamma$ collisions (see below) favor the $f_0(1500)$ to be the supernumerary non-$q\bar{q}$ state. According to [55, 60] the $f_0(1370)$ and $f_0(1710)$ would be dominantly $\bar{q}q$ states mixing with glue, while the $f_0(1500)$ would be dominantly a glueball mixing with $q\bar{q}$ states. Its suppressed $K\bar{K}$ decay would be due to interferences with the $f_0(1370)$ and $f_0(1710)$. In the analogous mixing scheme of

| Isospin | Decay channel | $\gamma^2$ |
|---|---|---|
| 0 | $\pi\pi$ | $3\cos^2\alpha$ |
| | $K\overline{K}$ | $(\cos\alpha - \sqrt{2}\sin\alpha)^2$ |
| | $\eta\eta$ | $(\cos\alpha\cos^2\phi - \sqrt{2}\sin\alpha\sin^2\phi)^2$ |
| | $\eta\eta'$ | $\frac{1}{2}\sin^2 2\phi\ (\cos\alpha + \sqrt{2}\sin\alpha)^2$ |
| 1 | $\eta\pi$ | $2\ \cos^2\phi$ |
| | $\eta'\pi$ | $2\ \sin^2\phi$ |
| | $K\overline{K}$ | $1$ |
| $\frac{1}{2}$ | $K\pi$ | $\frac{3}{2}$ |
| | $K\eta$ | $(\sin\phi - \frac{\cos\phi}{\sqrt{2}})^2$ |
| | $K\eta'$ | $(\cos\phi + \frac{\sin\phi}{\sqrt{2}})^2$ |

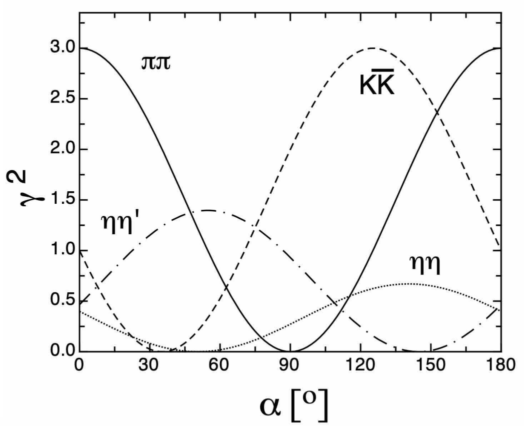

**Figure 63.2:** Left: SU(3) couplings $\gamma^2$ for scalar (or tensor) $q\bar{q}$ meson decays. The angles $\alpha$ and $\phi$ are defined as $\alpha = 54.7° + \theta$ and $\phi = 54.7° + \theta_P$, where $\theta$ is the mixing angle of the decaying isoscalars and $\theta_P$ the mixing angle in the $0^{++}$ ($2^{++}$) meson nonet. The partial decay width is given by $\Gamma = C \times \gamma^2 \times |F(q)|^2 \times q$, where $C$ is a nonet constant, $q$ the momentum of the decay products and $F(q)$ a form factor which may be taken as $|F(q)|^2 = q^{2\ell} \times \exp(-q^2/8\beta^2)$ with $\beta$= 0.5 GeV/c, and where $\ell$ is the relative angular momentum between the decay products ($\ell = 0$ for scalars and $\ell = 2$ for tensors). Right: SU(3) couplings as a function of mixing angle $\alpha$ for isoscalar decays for $\theta_P = -17.3°$ (from [55]).

Ref. [72], which uses central production data from WA102 and the hadronic $J/\psi(1S)$ decay data from BES [73, 74], glue is shared between the $f_0(1370)$ (mainly $n\bar{n}$), $f_0(1500)$(mainly glue) and $f_0(1710)$ (dominantly $s\bar{s}$). This agrees with the analyses [55, 60]. In the mixing scheme of Ref. [75] the QCD spectral sum rule also leads to a large gluonic component in the $f_0(1500)$ (satisfying the observed strong $\pi\pi$ and $f_0(500)f_0(500) \to 4\pi$ decays).

However, not everybody agrees that the strong $K\bar{K}$ signal is indicative of an $s\bar{s}$ structure for the $f_0(1710)$. The $f_0(1710)$ could still be the glueball, since the two-gluon coupling to $n\bar{n}$ appears to be suppressed by chiral symmetry [76], thus $K\bar{K}$ would be enhanced compared to $\pi\pi$. It was argued that chiral symmetry constraints in a multichannel analysis imply that the $f_0(1710)$ is an unmixed scalar glueball [77]. However, this view is challenged in [78].

The $K\bar{K}$ decay is also naturally enhanced in the extended linear sigma model with a dilaton as glueball [65] and in the holographic model of [66, 79] which both prefer the $f_0(1710)$ as the glueball. For a scalar glueball Ref. [66] finds a strong enhancement of the decays into $K\bar{K}$ and $\eta\eta$, in fairly close agreement with the measured branching ratios of the $f_0(1710)$, while Ref. [79] predicts the (so far not measured) rate into $\eta\eta'$ to be very small.

In $\gamma\gamma$ collisions leading to $K_S K_S$ [80] and $K^+K^-$ [81] a spin-0 signal is observed at the $f_0(1710)$ mass (together with a dominant spin-2 component), while the $f_0(1500)$ is not observed in $\gamma\gamma \to K\bar{K}$ nor $\pi^+\pi^-$ [82]. The $f_0(1500)$ is also not observed by Belle in $\gamma\gamma \to \pi^0\pi^0$, although a shoulder is seen which could also be due to the $f_0(1370)$ [83]. The absence of $f_0(1500)$ signal in the $\pi\pi$ channel in $\gamma\gamma$ collisions does not favor an $\overline{n}n$ interpretation for the $f_0(1500)$. The upper limit for $\Gamma_{2\gamma}$ ($<$ 1.4 keV) in $\pi^+\pi^-$ [12]) excludes a large $n\overline{n}$ content, and hence points to a mainly $s\overline{s}$ content [12], which contradicts the small $K\overline{K}$ decay branching ratio of the $f_0(1500)$ [18, 36, 84]. Belle finds that in $\gamma\gamma \to K_S K_S$ collisions the 1500 MeV region is dominated by the $f_2'(1525)$. The $f_0(1710)$ is also observed with a production rate × branching ratio compatible with an $s\bar{s}$ state [85]. Note, however, that the $\gamma\gamma$ couplings of glueballs are sensitive to glue mixing with $q\bar{q}$ [72]. Ref. [63] predicts the glueball $\gamma\gamma$ partial width in the few keV range (comparable to that of $q\bar{q}$ mesons) due to couplings of vector mesons to $\gamma$ via VDM.

Alternative assignments for the scalar glueball have been proposed: In Ref. [57] the gluonic signal is distributed over $f_0(1370)$, $f_0(1500)$ and another broad isoscalar around 1530 MeV, while the gluonic contribution to the $f_0(1710)$ is small. As mentioned already, in Ref. [47,65,66] the $f_0(1710)$ is the glueball state, as in Ref. [61], where the $f_0(1500)$ is the $q\bar{q}$ octet state degenerate with the $a_0(1450)$. In the generalized linear sigma model [86] the $a_0(980)$ is dominantly a tetraquark and $a_0(1450)$ a $q\bar{q}$ state. The $f_0(980)$, $f_0(1370)$ and $f_0(1500)$ (or $f_0(1710)$) are dominantly tetraquark, $q\bar{q}$ and glue.

In Ref. [59] the $f_0(500)$ and $f_0(1370)$ are signals from a single broad resonance proposed as the scalar glueball. The ground state scalar nonet then consists of the $f_0(980)$, $a_0(980)$, $K_0^*(1430)$, $f_0(1500)$ and $f_0(1710)$ [62]. The $f_0(980)$ and $f_0(1500)$ mix (similarly to the $\eta$ and $\eta'$ in the pseudoscalar nonet), while the $f_0(1500)$ mixes with a glueball in the 500 – 1000 MeV mass range, which is identified as the $f_0(500)$. A reanalysis of the CERN-Munich data shows no signal for the $f_0(1370)$ decaying into $\pi\pi$, in contrast to [54,87]. However, in this scheme the $K_0^*(700)$ and the $a_0(1450)$ are also left out.

In Ref. [88], a large $K^+K^-$ scalar signal reported by Belle in $B$ decays into $KK\bar{K}$ [89], compatible with the $f_0(1500)$, is explained as due to constructive interference of a flavor octet with a broad glueball background. However, the Belle data are inconsistent with the BaBar measurements which show instead a broad scalar at this mass for $B$ decays into both $K^\pm K^\pm K^\mp$ [90] and $K^+K^-\pi^0$ [91].

The $f_0(1500)$ is observed by BESII in $J/\psi(1S) \to \gamma\pi\pi$ [92] and by BESIII in $J/\psi(1S) \to \gamma\eta\eta$ [93] with a much smaller rate than for the $f_0(1710)$, which would speak against a glueball interpretation of the former, although the systematic errors are large. Also, the $f_0(1500)$ appears at a lower mass and the $f_0(1710)$ at a higher mass than the accepted values. However, the coupled channel analysis of more data on radiative $J/\psi(1S)$ decay from BESIII [54] (described in the next paragraph) finds comparable contributions from $f_0(1500)$ and $f_0(1710)$ with a preference for $\pi\pi$ decay over $K\bar{K}$ for the former and $K\bar{K}$ over $\pi\pi$ for the latter.

The authors of Ref. [54] have analyzed the high statistics data from $J/\psi(1S)$ radiative decays into $\pi^0\pi^0$, $K_S^0K_S^0$, $\eta\eta$ and $\omega\phi$ from BESIII, including data on $\pi\pi$, $\eta\eta$ and $\eta\eta'$ from the CERN SPS, BNL data on $\pi\pi \to K_S^0K_S^0$ and $\bar{p}p$ annihilation data from LEAR into various final states. The coupled channel analysis requires ten scalar mesons, the established ones in the *Review* and the four isoscalars $f_0(2020, 2100, 2200, 2330)$ so far omitted from the *Summary Table.* As a new feature, the $f_0(1710)$ splits into two states, one at about 1700 MeV, the former $f_0(1710)$, and a new one around 1770 MeV being mandatory to obtain a good fit. A broad (∼370 MeV) enhancement is observed in $J/\psi(1S)$ radiative decays around 1865 MeV, attributed to the contribution of glue distributed among the scalar mesons, with a very strong contribution from the new 1770 MeV state [94]. In this model there are no supplementary states as the glue ‘fragments’ between the various isoscalars.

Ref. [95] reports on a very recent re-analysis of the BESIII $J/\psi(1S)$ radiative decay data into $\pi^0\pi^0$ and $K_S^0K_S^0$. A good fit is obtained with only two scalar resonances below 2 GeV, the $f_0(1500)$ and $f_0(1710)$. The latter couples much more strongly to both channels, which points to a sizeable glueball component.

### 63.3.2 *Tensor glueballs*

Above the well known $f_2(1270)$ and $f_2'(1525)$ $q\bar{q}$ mesons, the $f_2(1640)$ and $f_2(1950)$ are tentatively assigned to the $2^3P_2$ nonet (see Fig. 63.1). The broad $f_2(1950)$ has been observed by several experiments, *e.g.* in central production [19] and in $\bar{p}p$ annihilation in flight [96] and is often identified as the ground state of the pomeron [97,98]. Three further isoscalar tensors are established, the $f_2(2010)$, $f_2(2300)$ and $f_2(2340)$, which are in the range of the $1^3F_2$ and $3^3P_2$ nonets, and in the expected region for the $2^{++}$ glueball. The large $\phi\phi$ cross section in $\overline{p}p$ just above threshold [99] could be due to the production of the $2^{++}$ glueball, in accord with earlier observations in $\pi^- N$ reactions [100, 101] and in central collisions [102].

The $f_2(2010)$, $f_2(2300)$ and $f_2(2340)$ have been observed by BESIII in $J/\psi(1S) \to \gamma\phi\phi$ [103]. The production rate of a tensor glueball in $J/\psi(1S)$ radiative decay has been calculated in quenched lattice QCD to be large (around 1%) [104] and is claimed by BESIII [103] to be compatible with their rates measured in $J/\psi(1S) \to f_2(2340)$, $f_2(2340) \to \eta\eta$ [93], and $f_2(2340) \to \phi\phi$ [103]. The relatively narrow $f_2(2300)$ with a measured width of 149 MeV [101] is preferred by the holographic QCD model of Ref. [105]. However, the tensor glueball is predicted to be much broader (600-900 MeV) by the holographic model of Ref. [106].

There is no evidence for a narrow meson, $f_J(2220)$ (a tensor candidate) in $\overline{p}p$ annihilation (see the note under the $f_J(2220)$ in the 2004 issue of the *Review*). The measured partial width to $\overline{p}p$ in radiative $J/\psi(1S)$ decay [107] is too large and inconsistent with the upper limit from $\overline{p}p$ annihilation into $\pi\pi$ [108].

## 63.4 Pseudoscalar mesons

We now deal with the first radial excitations of the $0^{-+}$ nonet (Fig. 63.1). The $\pi(1300)$ is a very broad resonance (200–600 MeV) decaying into $3\pi$. The $K(1460)$ was recently confirmed with high statistics data from LHCb in $D^0 \to K3\pi$ decays [109] and has become an established kaon excitation. The first observation of an isoscalar resonance around 1425 MeV – the $E$-meson – was made in $p\overline{p}$ annihilation at rest into $E\,\pi^+\pi^-$, $E \to K\overline{K}\pi$ [110]. This state was reported to decay into $a_0(980)\pi$ and $K^*(892)\overline{K}$ with roughly equal contributions. An isoscalar state, the $\iota$ meson, was observed in radiative $J/\psi(1S)$ decay into $K\overline{K}\pi$ [111–113] and $\gamma\rho$ [114], and was considered at that time as a glueball candidate, owing to its strong signal in radiative $J/\psi(1S)$ decay. The $E$ and $\iota$ were later assumed to be the same state, called $\eta(1440)$.

The $\eta(1295)$ has been observed in various production mechanisms, in $\pi^- p$ experiments [115–118], and in $\overline{p}p$ annihilation [84, 119, 120]. In $J/\psi(1S)$ radiative decay, the $\eta(1295)$ signal is also evident in the $0^{-+}$ $\eta\pi\pi$ wave of the DM2 data [121]. Also BaBar reports a signal around 1295 MeV in $B$ decays into $\eta\pi\pi K$ [122]. Let us therefore assume in the following that the $\eta(1295)$ is one of the isoscalars in this nonet.

However, two isoscalars were later observed in this mass region, the $\eta(1405)$ and $\eta(1475)$. The former decays mainly into $a_0(980)\pi$ (or direct $K\overline{K}\pi$) and the latter mainly into $K^*(892)\overline{K}$. The simultaneous observation of two pseudoscalars is reported in three production mechanisms: $\pi^- p$ [115, 123], radiative $J/\psi(1S)$ decay [121, 124], and $\overline{p}p$ annihilation at rest [125–128]. All of them give values for the masses, widths, and decay modes that are in reasonable agreement. (However, Ref. [121] favors a state decaying into $K^*(892)\overline{K}$ at a lower mass than the state decaying into $a_0(980)\pi$.) In $J/\psi(1S)$ radiative decay, the $\eta(1405)$ decays into $K\overline{K}\pi$ through $a_0(980)\pi$, and hence a signal is also expected in the $\eta\pi\pi$ mass spectrum. This was indeed observed by MARK III in $\eta\pi^+\pi^-$ [129], which reported a mass of 1400 MeV, in line with the existence of the $\eta(1405)$ decaying into $a_0(980)\pi$. Two states were also reported by BES: Around 1452 MeV BESII observed a $K\bar{K}\pi$ enhancement in $J/\psi(1S) \to \omega K\bar{K}\pi$ but not in $J/\psi(1S) \to \phi K\bar{K}\pi$ [130], while BESIII reported a 52 MeV broad state in $\psi(2S) \to \omega K^* K$ [131], both left without $J^{PC}$ determination.

The $K\overline{K}\pi$ and $\eta\pi\pi$ channels were studied in $\gamma\gamma$ collisions by L3 [132]. (For the $2\gamma$ couplings of glueballs and $q\bar{q}$ mesons see [63, 133, 134].) The analysis led to a clear $\eta(1475)$ signal in $K\overline{K}\pi$, decaying into $K^*\overline{K}$, well identified in the untagged data sample, where spin 1 resonances are not allowed. At the same time, L3 did not observe the $\eta(1405)$, neither in $K\overline{K}\pi$ nor in $\eta\pi\pi$ [132]. On the other hand, CLEO-II did not observe any pseudoscalar signal with tagged $\gamma$'s in $\gamma\gamma \to \eta(1475) \to K^0_S K^\pm\pi^\mp$ [135], with an upper limit slightly smaller than the signal observed by

L3. After the CLEO-II result L3 performed a further analysis with full statistics [136], confirming their previous evidence for the $\eta(1475)$. The CLEO upper limit [135] for $\Gamma_{\gamma\gamma}$, and the L3 results [136], are consistent with the world average for the $\eta(1475)$ width. BaBar [122] also reported the $\eta(1475)$ in $B$ decays into $K\bar{K}^*$ recoiling against a $K$, while upper limits only were given for the $\eta(1405)$.

Hence, in radiative $J/\psi(1S)$ decay, $\pi^- p$ and $\overline{p}p$ annihilation at rest two isoscalar signals are observed in the 1400 – 1500 MeV mass region, while the $\eta(1405)$ is not seen in $\gamma\gamma$ interactions nor in $B$ decays. The $\eta(1475)$ could be the first radial excitation of the $\eta'$, with the $\eta(1295)$ being the first radial excitation of the $\eta$. Ideal mixing, suggested by the $\eta(1295)$ and $\pi(1300)$ mass degeneracy, would then imply that the second isoscalar in the nonet is mainly $s\overline{s}$, and hence couples to $K^*\overline{K}$, in agreement with properties of the $\eta(1475)$. Also, its width agrees with the expected one for the radially excited $s\overline{s}$ state [64, 69]. A study of radial excitations of pseudoscalar mesons [137] also favors the $s\overline{s}$ interpretation of the $\eta(1475)$. However, due to the strong kinematical suppression in $\eta(1405) \to K^*\overline{K}$ the data are not sufficient to exclude a sizeable $s\overline{s}$ admixture also in the $\eta(1405)$.

The supernumerary isoscalar $\eta(1405)$ would be a candidate for the $0^{-+}$ glueball in the fluxtube model [138], in which the $0^{++}$ glueball is also naturally related to a $0^{-+}$ glueball with mass degeneracy broken in QCD. However, this scenario is not favored by lattice gauge theories which predict the $0^{-+}$ state above 2 GeV [48, 139] (see 'Quark model' in this issue of the *Review*). Nevertheless, the pseudoscalar glueball could lie at a lower mass than predicted from lattice calculation [140], see also Refs. [141–143]. A detailed review of the experimental situation on the pseudoscalar glueball is available in Ref. [144].

Here also, there are alternative explanations. The mere existence of the $\eta(1295)$ is questioned in Refs. [3, 145], in which the authors also propose a single pseudoscalar meson at 1440 MeV, the first radial excitation of the $\eta$. According to Ref. [3] the splitting of the 1440 MeV state into $\eta(1405)$ and $\eta(1475)$ is due to nodes in the decay amplitudes, which differ in $\eta\pi\pi$ and $K^*(892)\overline{K}$. The splitting could also be due to a triangle singularity, hence the manifestation of one state only (Fig. 63.3) [146–148]. In Ref. [149], using the triangle singularity approach of [146], the authors conclude that the BESIII results can be reproduced either with the $\eta(1405)$ or the $\eta(1475)$, or by a mixture of these two states.

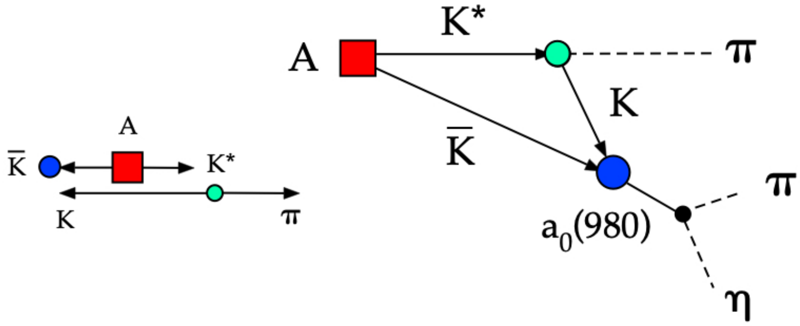

**Figure 63.3:** Triangle singularity: a state $A$ decays into $K^*\overline{K}$. The $K$ from $K^*$ decay catches up with the $\overline{K}$ and excites the $a_0(980)$ resonance which in turn decays into $\eta\pi$. This mechanism can lead to two distinct peaks (depending on the width of $A$), one in $K^*\overline{K}$ and the other via rescattering in $\eta\pi\pi$. Similarly, the virtual $K^*\overline{K}$ loop can couple to $f_0(980)$ which, decaying into $\pi\pi$, leads to a peak in the $3\pi$ final state.

To summarise this section, the experimental data on the 1400 — 1500 MeV region span several decades, various production mechanisms and decay modes, with models for data analysis evolving with time. A comprehensive coherent picture of all available data is therefore difficult. We believe that there is sufficient evidence to consider the $0^{-+}$ nonet with the $\eta(1440)$ in fig. 63.1 as established. Whether one or two different states – $\eta(1405)$ and $\eta(1475)$ – exist is an open question, in which case the $\eta(1405)$ would be supernumerary. There is a wide number of experimental results indicating the presence of two separate states but, as mentioned above, data are also consistent with one state only. Theoretical interpretations of the most recent data are not able to lift the ambiguity.

## 63.5 Vector mesons

In this section we restrict ourselves to the more interesting isovector spectrum which contains broad and overlapping states.

### 63.5.1 *The* $\rho(770)$ *meson*

The determination of the parameters of the $\rho(770)$ is beset with many difficulties because of its large width. The line shape depends on the production process and is not described by a relativistic Breit-Wigner function with a $P$-wave width, but requires some additional shape parameter. This dependence on parameterization was demonstrated long ago [150]. Bose-Einstein correlations are another source of shifts in the $\rho(770)$ line shape, particularly in multiparticle final-state systems [151].

The same model dependence afflicts any other source of resonance parameters, such as the energy dependence of the phase shift $\delta_1^1$, or the pole position. It is, therefore, not surprising that a study of $\rho(770)$ dominance in the decays of the $\eta$ and $\eta'$ reveals the need for specific dynamical effects, in addition to the $\rho(770)$ pole [152, 153].

The cleanest determination of the $\rho(770)$ mass and width comes from $e^+e^-$ annihilation and $\tau$-lepton decays. ALEPH data [154] showed that the charged $\rho(770)$ parameters measured from $\tau$-lepton decays are consistent with those of the neutral one determined from $e^+e^-$ data [155]. This conclusion is qualitatively supported by the later studies of CLEO [156] and Belle [157]. However, a comparison of the two-pion mass spectrum in $\tau$ decays from OPAL [158], CLEO [156], and ALEPH [159, 160], and the $e^+e^- \to \pi^+\pi^-$ cross section from CMD-2 [161, 162], showed significant discrepancies between the two shapes which can be as high as 10% above the $\rho$ meson mass [163,164]. This discrepancy remains after measurements of the two-pion cross section in $e^+e^-$ annihilation at KLOE [165–168], SND [169, 170], BaBar [171] and, more recently BESIII [172]. The effect is not accounted for by isospin breaking [173–176], but the accuracy of its calculation may be overestimated [177, 178].

This problem seems to be solved after a recent analysis in [179], which showed that after correcting the $\tau$ data for the missing $\rho$–$\gamma$ mixing contribution, besides the other known isospin symmetry violating corrections, the $\pi\pi$ isospin 1 part of the hadronic vacuum polarization contribution to the muon $g - 2$ is fully compatible between $\tau$ based and $e^+e^-$ based evaluations. The global fit of the whole set of the $\rho$, $\omega$, and $\phi$ decays, taking into account mixing effects in the hidden local symmetry model, also showed consistency of the data on $\tau$ decays to two pions and $e^+e^-$ annihilation [180, 181]. However, because of the progress in $e^+e^-$ data, the $\tau$ input is now less precise and less reliable due to additional theoretical uncertainties [182] decreasing the importance of $\tau$ decay for the determination of the $\rho(770)$ parameters.

### 63.5.2 *The* $\rho(770)$ *excitations*

In our 1988 edition, we replaced the $\rho(1600)$ entry with two new ones, the $\rho(1450)$ and the $\rho(1700)$, because there was emerging evidence that the 1600-MeV region actually contains two $\rho$-like resonances. This possibility was pointed out by a theoretical analysis [183] on the consistency of the $2\pi$ and $4\pi$ electromagnetic form factors and the $\pi\pi$ scattering length. A consistent picture of $e^+e^- \to 2\pi, 4\pi$ and diffractive photoproduction is obtained with two resonances [184]. The existence of the $\rho(1450)$ was supported by the analysis of $\eta\rho^0$ mass spectra obtained in photoproduction and $e^+e^-$ annihilation [185], as well as that of $e^+e^- \to \omega\pi$ [186].

The analysis of [184] was further extended by [187, 188] to include new data on $4\pi$-systems produced in $e^+e^-$ annihilation, and in $\tau$ decays ($\tau \to 4\pi$ and $e^+e^- \to 4\pi$ are related by the Conserved Vector Current hypthesis). These systems were successfully analyzed using interfering contributions from two $\rho$-like states, and from the tail of the $\rho(770)$ two-body decay. While specific conclusions on $\rho(1450) \to 4\pi$ were obtained, little could be said about the $\rho(1700)$. Independent evidence for two $1^-$ states is provided by [189] in $4\pi$ electroproduction at $\langle Q^2 \rangle = 1$ $(\mathrm{GeV}/c)^2$, and by [190] in a high-statistics sample of the $\eta\pi\pi$ system in $\pi^-p$ charge exchange.

This scenario with two overlapping resonances is supported by other data. DM2 [191] measured the pion form factor in the interval 1.35–2.4 GeV, and observed a deep minimum around 1.6 GeV. The best fit was obtained with the hypothesis of $\rho$-like resonances at 1420 and 1770 MeV, with widths of about 250 MeV. DM2 [192] found that the $e^+e^- \to \eta\pi^+\pi^-$ cross section is better fitted with two fully interfering Breit-Wigners, with parameters in fair agreement with those of [184] and [191]. These results can be considered as a confirmation of the $\rho(1450)$.

Decisive evidence for the $\pi\pi$ decay mode of both $\rho(1450)$ and $\rho(1700)$ comes from $\overline{p}p$ annihilation at rest [193]. It has been shown that these resonances possess a $K\overline{K}$ decay mode [84, 194, 195]. This decay mode has also been studied in three-body hadronic $B$ decays [196]. High-statistics studies of the decays $\tau \to \pi\pi\nu_\tau$ [154, 156], and $\tau \to 4\pi\nu_\tau$ [197] also require the $\rho(1450)$, but are not sensitive to the $\rho(1700)$, being too close to the $\tau$ mass. A recent very-high-statistics study of the $\tau \to \pi\pi\nu_\tau$ from Belle [157] reports the first observation of both $\rho(1450)$ and $\rho(1700)$ in $\tau$ decays. A clear picture of the two $\pi^+\pi^-$ resonances interfering with the $\rho(770)$ in $e^+e^-$ annihilation was also reported by BaBar using the ISR (Initial State Radiation) method [198].

The structure of these $\rho$ states is not yet completely clear. The authors of Refs. [64, 199] claim that $\rho(1450)$ has a mass consistent with radial $2S$, but its decays show characteristics of hybrids, and suggest that this state may be a $2S$-hybrid mixture. Hybrid states could have a $4\pi$ decay mode dominated by the $a_1\pi$ [200]. Such behavior has been observed by [201] in $e^+e^- \to 4\pi$ in the energy range 1.05–1.38 GeV, and by [197] in $\tau \to 4\pi$ decays. CLEO [202] and Belle [203] observe the $\rho(1450) \to \omega\pi$ decay mode in $B$-meson decays, however, do not find $\rho(1700) \to \omega\pi^0$. A similar conclusion is made by [204, 205], who studied the process $e^+e^- \to \omega\pi^0$ and do not observe a statistically significant signal of the $\rho(1700)$. Various decay modes of the $\rho(1450)$ and $\rho(1700)$ are observed in $\overline{p}n$ and $\overline{p}p$ annihilation [206, 207], but no definite conclusions can be drawn. More data should be collected to clarify the nature of the $\rho$ states, particularly in the energy range above 1.6 GeV.

We now list under a separate entry the $\rho(1570)$, the $\phi\pi$ state with $J^{PC} = 1^{--}$ earlier observed by [208] – referred to as $C(1480)$) – and recently confirmed by [209] and [210]. While [211] shows that it may be a threshold effect, Refs. [187] and [212] suggest two independent vector states with this decay mode. The $C(1480)$ has not been seen in the $\overline{p}p$ [213] and $e^+e^-$ [214, 215] experiments. However, the sensitivity of the two latter is an order of magnitude lower than that of [209]. Note that [209] can not exclude that their observation is due to an OZI-suppressed decay mode of the $\rho(1700)$.

Several observations on the $\omega\pi$ system in the 1200 MeV region [216–222] may be interpreted in terms of either $J^P = 1^-$ $\rho(770) \to \omega\pi$ production [223], or $J^P = 1^+$ $b_1(1235)$ production [221, 222]. We argue that no special entry for a $\rho(1250)$ is needed. The LASS amplitude analysis [224] showing evidence for $\rho(1270)$ is preliminary and needs confirmation. However, evidence for the $\rho(1250)$ from a reanalysis of elastic scattering data is claimed in Ref. [225]. For completeness, the relevant observations are listed under the $\rho(1450)$.

Recently Ref. [226] reported a very broad $1^{--}$ resonance-like $K^+K^-$ state in $J/\psi \to K^+K^-\pi^0$ decays. Its pole position corresponds to mass of 1576 MeV and width of 818 MeV. Its exotic structure (molecular or multiquark) is suggested [227–229], while in Refs. [230] and [231] this is explained by the interference between the $\rho(1450)$ and $\rho(1700)$. The latter statement is qualitatively supported by BaBar [232] and SND [233]. We quote [226] as $X(1575)$ in the section "Further States."

Evidence for $\rho$-like mesons decaying into $6\pi$ was first noted by [234] in the analysis of $e^+e^- \to 6\pi$ [235, 236] and diffractive photoproduction [237]. The authors of Ref. [234] argued that two states at about 2.1 and 1.8 GeV exist: while the former is a candidate for the $\rho(2150)$, the latter could be a manifestation of the $\rho(1700)$ distorted by threshold effects. BaBar reported observations of the new decay modes of the $\rho(2150)$ in the channels $\eta'(958)\pi^+\pi^-$, $f_1(1285)\pi^+\pi^-$ [238] and $\pi^+\pi^-$ [239]. The decay of the $\rho(2150)$ into $K^+K^-$ has been observed by BESIII [240] and confirmed by BaBar [241]. The relativistic quark model [242] predicts the $2^3D_1$ state with $J^{PC} = 1^{--}$ at 2.15 GeV which can be identified with the $\rho(2150)$.

Under $\rho(1900)$ we list various observations of irregular behavior of the cross sections near the $N\bar{N}$ threshold. Dips of various width around 1.9 GeV were reported by the E687 Collaboration (a narrow one in the $3\pi^+3\pi^-$ diffractive photoproduction [243, 244]), by the FENICE experiment (a narrow structure in the $R$ value [245]), by BaBar in ISR (a narrow structure in $e^+e^- \to \phi\pi$ final state [209], but much broader in $e^+e^- \to 3\pi^+3\pi^-$ and $e^+e^- \to 2(\pi^+\pi^-\pi^0)$ [246]), by CMD-3 (also a rather broad dip in $e^+e^- \to 3\pi^+3\pi^-$ [247]). A dedicated scan of the $N\bar{N}$-threshold region by CMD-3 confirms this effect in the $e^+e^- \to 3\pi^+3\pi^-$ and $e^+e^- \to K^+K^-\pi^+\pi^-$ final states, but does not see it in the cross section of $e^+e^- \to 2\pi^+2\pi^-$ [248]. Most probably, these structures emerge as a threshold effect due to the opening of the $N\bar{N}$ channel [249–251]. A similar enhancement is observed by BESIII in $e^+e^- \to \Lambda\bar{\Lambda}$ near the $\Lambda\bar{\Lambda}$ threshold [252].

## 63.6 Axial-vector mesons

The $J^{PC} = 1^{++}$ nonet consists of the isovector $a_1(1260)$, the isoscalars $f_1(1285)$ and $f_1(1420)$, and the $K_{1A}$, which is a superposition of the physical states $K_1(1270)$ and $K_1(1400)$ with a mixing angle of about 35° [253]. The nonet mixing angle $\theta_{1^{++}}$ is around 23°. The orthogonal combination – the $K_{1B}$ – belongs to the $1^{+-}$ nonet with a mixing angle $\theta_{1^{+-}}$ of about 28° [253].

The mass region above 1400 MeV is rather complex [254–256]. The $f_1(1420)$ was first reported in $\pi^-p$ reactions at 4 GeV/$c$, decaying into $K^*\overline{K}$ [257]. The $f_1(1420) \to K\overline{K}\pi$ was also observed in a reanalysis of the MARK III data [124], and a $C$=+1 state seen in tagged $\gamma\gamma$ collisions [258]. Axial-vector mesons are not observed in $\overline{p}p$ annihilation at rest in liquid hydrogen, which proceeds dominantly through $S$-wave annihilation. However, in gaseous hydrogen, $P$-wave annihilation is enhanced and the $f_1(1420)$ is indeed observed, decaying into $K^*\overline{K}$ [126]. The $f_1(1420)$, decaying into $K\overline{K}\pi$, is also seen in $pp$ central production, together with the $f_1(1285)$. The latter decays via $a_0(980)\pi$, and the former only via $K^*\overline{K}$. The $K^0_SK^0_S\pi^0$ decay mode of the $f_1(1420)$ establishes unambiguously $C$=+1. Neither central production [259] nor $\pi^-p$ interactions at 100 GeV [117] find any evidence for a $\eta\pi\pi$ decay mode of the $f_1(1420)$.

The $f_1(1285)$ has been suggested to be a $K^*\overline{K}$ molecule [260]. However, LHCb has determined the $1^{++}$ nonet mixing angle to be consistent with a mostly $n\bar{n}$ structure for the $f_1(1285)$ from $\overline{B}^0/\overline{B}^0 \to J/\psi(1S)f_1(1285)$, independent of the identity of its isoscalar partner [261]. The mixing angle $\theta_{1^{++}} = 24°$ agrees with that of Ref. [253] quoted above. The ratio of $\overline{B}^0/\overline{B}^0_s$ decay rates also excludes the tetraquark interpretation of the $f_1(1285)$ [261]. This is consistent with earlier determinations assuming that the $f_1(1420)$ is the other isoscalar in the nonet [262].

A resonance candidate decaying into $3\pi$ had been reported earlier by COMPASS at 1420 MeV in $\pi^-p \to \pi^-\pi^-\pi^+p$ [263, 264]. The signal appeared as a clear peak in the $1^{++}$ $f_0(980)\pi$ $P$-wave and had the expected phase motion of a new resonance, called the $a_1(1420)$. However, the signal appears to originate from a triangle singularity: the decay of the $a_1(1260)$ into $K^*(\to K\pi)\bar{K}$ is followed by rescattering into the $f_0(980)\pi$ channel, $f_0(980) \to \pi\pi$ (see the caption of fig. 63.3) [265]. Accordingly, this entry has been removed from the *Listings* in the *Review*.

A similar triangle singularity is proposed for the $f_1(1420)$, resulting from the $K^*\overline{K}$ and $a_0(980)\pi$ decay modes of the $f_1(1285)$ [266]. The $f_1(1420)$ was also suggested to be a hybrid $q\bar{q}g$ meson [267] or a $K^*\overline{K}$ molecule, due to the proximity of the $K^*\overline{K}$ threshold [268]. Indeed, the $f_1(1420)$ is not seen in $K^-p$ [269], which argues against the $f_1(1420)$ being the $s\bar{s}$ member of the $1^{++}$ nonet. In this case the $s\bar{s}$ partner of the $f_1(1285)$ could be the $f_1(1510)$ which is, however, not well established [270].

The $f_1(1510)$ was seen to decay into $K\bar{K}\pi$ in $K^-p$ interactions at 4 and 11 GeV/c, recoiling against a $\Lambda$ [269, 271], which points to an $s\bar{s}$ state. Evidence was also reported in $\pi^-p$ interactions, based on the phase motion of the $1^{++}$ $K^*\overline{K}$ wave [256]. A somewhat broader $1^{++}$ signal is observed in $J/\psi(1S) \to \gamma\eta\pi^+\pi^-$ [272] and a small signal in $J/\psi(1S) \to \gamma\eta'\pi^+\pi^-$ [273]. However, there is no evidence for the $f_1(1510)$ in other $K^-p$ experiments [274,275], while the $f_1(1420)$ is observed in $K^-p$ but not in $\pi^-p$ [274]. The $f_1(1510)$ is not seen in central collisions [276], nor in $\gamma\gamma$ collisions [277], although surprisingly for an assumed $s\bar{s}$ meson, a signal is reported in $4\pi$ decays [278]. Given this confusing experimental situation, the meson classification in the $1^{++}$ nonet is not entirely settled.

## 63.7 Hybrid mesons

Hybrids may be viewed as $q\bar{q}$ mesons with a vibrating gluon flux tube. In contrast to glueballs, they can have isospin 0 or 1 and be electrically charged. The mass spectrum of hybrids with exotic (non-$q\bar{q}$) quantum numbers was predicted in Ref. [279], while Ref. [280] also deals with non-exotic quantum numbers. The ground-state hybrids with quantum numbers ($0^{-+}$, $1^{-+}$, $1^{--}$, and $2^{-+}$) are expected around 1.7 to 1.9 GeV. Lattice calculations predict that the hybrid with exotic quantum numbers $1^{-+}$ lies at a mass of $1.9 \pm 0.2$ GeV [281, 282]. Most hybrids are expected to be rather broad, but some can be as narrow as 100 MeV [283]. They prefer to decay into a pair of $S$- and $P$-wave mesons. The lattice study in [284, 285], based on full QCD with pion masses around 400 MeV, finds that several of the high-lying states observed in their spectrum show significant overlap with gluon rich source terms interpreted as hybrid states. A very broad $1^{-+}$ structure is predicted by a recent lattice calculation [286]. The main decay mode is $b_1(1235)\pi$ (a pair of $S$- and $P$-wave mesons), while other modes, such as $\rho\pi$, $\eta\pi$ and $\eta'\pi$, are suppressed by at least an order of magnitude. For an experimental and theoretical review on hybrid mesons see [4, 287].

There are currently two $1^{-+}$ candidates, the $\pi_1(1400)$ and $\pi_1(1600)$. The ∼350–400 MeV broad $\pi_1(1400)$, was reported in $\pi^-p \to \eta\pi^-p$ [288, 289] and in $\pi^-p \to \eta\pi^0 n$ [290]. It was observed as an interference between the angular momentum $L = 1$ and $L = 2$ $\eta\pi$ amplitudes, leading to a forward/backward asymmetry in the $\eta\pi$ angular distribution. This state had been reported earlier in $\pi^-p$ reactions [291], but ambiguous solutions in the partial wave analysis were pointed out [292, 293]. A resonating $1^{-+}$ contribution to the $\eta\pi$ $P$-wave is also required in the Dalitz plot analysis of $\overline{p}n$ annihilation into $\pi^-\pi^0\eta$ [294], and in $\overline{p}p$ annihilation into $\pi^0\pi^0\eta$ [295]. Mass and width are consistent with the results of Ref. [288]. A coupled channel re-analysis of the $\pi^0\pi^0\eta$, $\pi^0\eta\eta$ and $K^+K^-\pi^0$ Crystal Barrel data at 900 MeV/c, supplemented with data from other collaboration in $\pi\pi \to \pi\pi, K\bar{K}, \eta\eta$ and $\eta\eta'$, leads to a single $1^{-+}$ ∼600 MeV broad state with a pole around 1400 MeV, decaying into $\eta\pi^0$ [296]. Ref. [297] suggested for the production in $\pi^-p$ that a Deck-generated $\eta\pi$ background from final state rescattering in $\pi_1(1600)$ decay could mimic $\pi_1(1400)$, a mechanism that is, however, absent in $\overline{p}p$ annihilation.

The $\pi_1(1600)$, decaying into $\rho\pi$, was reported by COMPASS with 190 GeV pions hitting a lead target [298, 299]. It had already been observed in $\pi^-p$ interactions in the decay modes $\eta'\pi$ [300],

$f_1(1285)\pi$ [301], and $\omega\pi\pi$ [302], $b_1(1235)\pi$, but not $\eta\pi$ [303]. A strong enhancement in the $1^{-+}$ $\eta'\pi$ wave, compared to $\eta\pi$, was reported at this mass in [304]. The enhancement is also observed by COMPASS in $\pi^- p \to \eta'\pi$ but in $\eta\pi$ a peak appears at 1400 MeV and in $\eta'\pi$ at 1600 MeV [305]. A coupled channel analysis of the $\eta\pi$ and $\eta'\pi$ COMPASS data leads to a single pole at 1564 ± 89 MeV, with a width of 492 ± 115 MeV [306]. Furthermore, a combined analysis of the COMPASS and Crystal Barrel data at 900 MeV/c leads to compatible results, a single pole around 1623 MeV, with a width of about 455 MeV, although a two-pole scenario cannot be completely excluded [307]. The predicted width from the lattice calculation [286] is compatible with the COMPASS result.

Hybrid candidates with the non-exotic quantum numbers $0^{-+}$, $1^{--}$, and $2^{-+}$ have also been reported: the $\pi(1800)$ is somewhat narrow if interpreted as the second radial excitation of the pion. It decays mostly into a pair of $S$- and $P$-wave mesons [308, 309], in line with expectations for $0^{-+}$ hybrid mesons. The evidence for $1^{--}$ hybrids in $e^+e^-$ annihilation and in $\tau$ decays has been discussed in [200]. The near degeneracy of the $\eta_2(1645)$ and $\pi_2(1670)$ suggests ideal mixing in the $2^{-+}$ $q\bar{q}$ nonet, and hence, the second isoscalar, presumably the $\eta_2(1870)$, should be mainly $s\bar{s}$. However, the $\eta_2(1870)$ is also observed in $\bar{p}p$ annihilation and decays mainly into $a_2(1320)\pi$ and $f_2(1270)\pi$ [310], with a relative rate compatible with a hybrid state [280]. Evidence for another exotic $\pi_1(2015)$ has been claimed in $\pi^- p$ interactions [301, 302].

Summarizing, there is evidence for a very broad $1^{-+}$ enhancement in the 1400–1600 MeV region which consists in one or perhaps two exotic states, the lowest one seemingly favored by $\bar{p}p$ annihilation data, the highest one by high energy $\pi^- p$ data. As isovectors, $\pi_1(1400)$ and $\pi_1(1600)$ cannot be glueballs. The coupling to $\eta\pi$ of the former points to a four-quark state [311], while the $\eta'\pi$ coupling of the latter is favored for hybrid states [312, 313]. The mass of $\pi_1(1600)$ is not far below the lattice and flux tube model predictions.

## 63.8 Tetraquark states

The existence of multiquark states was suggested a long time ago, based on duality arguments [314, 315]. The most prominent tetraquark candidates are the $a_0(980)$ and $f_0(980)$ [316–318]. A remarkable prediction for the existence of low-lying four quarks states is based on color hyperfine splitting. The lowest ground state ($L = 0$) tetraquark multiplet is predicted to be a scalar nonet. The scalar nonet lies just below 1 GeV when one assumes as mass scale the hyperfine color splitting between the $\rho$ and the $\pi$ (for a simple derivation see [8]). Assuming the classification in Table 63.1 one then gets the nonet structures

$$\begin{aligned}
|a_0(980)\rangle &= |us\bar{d}\bar{s}\rangle, \frac{1}{\sqrt{2}}|(u\bar{u} - d\bar{d})s\bar{s}\rangle, |\bar{u}\bar{s}ds\rangle, \\
|f_0(980)\rangle &= \frac{1}{\sqrt{2}}|(u\bar{u} + d\bar{d})s\bar{s}\rangle, \\
|f_0(500)\rangle &= |\bar{u}\bar{d}ud\rangle, \\
|K_0^*(700)\rangle &= |\bar{s}\bar{d}ud\rangle, |\bar{s}\bar{u}ud\rangle \quad \text{and} \quad |\bar{u}\bar{d}us\rangle, |\bar{u}\bar{d}ds\rangle.
\end{aligned}$$

The two isoscalars are expected to mix with the angle $\varphi$:

$$\begin{aligned}
|f_0(980)\rangle &= \cos\varphi|s\bar{s}\rangle + \sin\varphi|n\bar{n}\rangle, \\
|f_0(500)\rangle &= -\sin\varphi|s\bar{s}\rangle + \cos\varphi|n\bar{n}\rangle. \qquad (63.1)
\end{aligned}$$

Whether these mesons are really tetraquark states or $q\bar{q}$ mesons, is still an open issue. The $f_0(980)$ is strongly produced in $D_s^+$ decay [320], which suggests a large $s\bar{s}$ component, due to the Cabibbo-favored $c \to s$ coupling. However, the mainly $n\bar{n}$ $f_0(1370)$ is also strongly produced in $D_s^+$ decay, hence additional graphs must contribute [321].

**Table 63.2:** Coupling amplitudes for $\bar{B}^0$ and $\bar{B}^0_s$ decays into $J/\psi(1S)f_0(500)/f_0(980)$, depending on the $q\bar{q}$ or tetraquark structures of the $f_0(500)$ and $f_0(980)$ (from [319] where illustrative decay diagrams can be found). The angle $\varphi$ is defined in Eq. 63.1.

| | $\bar{B}^0$ | | $\bar{B}^0_s$ | |
|---|---|---|---|---|
| | $f_0(980)$ | $f_0(500)$ | $f_0(980)$ | $f_0(500)$ |
| $q\bar{q}$ | $\sin\varphi/\sqrt{2}$ | $\cos\varphi/\sqrt{2}$ | $\cos\varphi$ | $\sin\varphi$ |
| $qq\bar{q}\bar{q}$ | $1/\sqrt{2}$ | 1 | $\sqrt{2}$ | 0 |

The relative branching ratios for $\bar{B}^0$ and $\bar{B}^0_s \to J/\psi(1S)f_0(500)$ and $J/\psi(1S)f_0(980)$ can be used to probe the $q\bar{q}$ or tetraquark natures of the $f_0(500)$ and $f_0(980)$, as proposed in Refs. [319, 322]. LHCb observes the $f_0(980)$ in $\bar{B}^0_s$ decays, but not the $f_0(500)$ [26], as would be expected for tetraquarks (see Table 63.2). In contrast, LHCb also observes the $f_0(500)$ in $\bar{B}^0$ decays, but not the $f_0(980)$ with an upper limit eight standard deviations below the predicted value for a tetraquark state [27]. However, these contradicting findings have been challenged by a dispersive analysis [323], using a model independent inclusion of hadronic final state interactions, in which a substantial $f_0(980)$ contribution is nevertheless found in $\bar{B}^0$-decays, thus still leaving the tetraquark structure as an open possibility.

The $f_0(980)$ and $a_0(980)$ could also be $K\overline{K}$ molecular states [324–326], being close to the $K\overline{K}$ threshold and decaying strongly into $K\overline{K}$. For $q\bar{q}$ states, the expected $\gamma\gamma$ widths [327, 328] are not significantly larger than for molecular states [327, 329] and both predictions are consistent with data. Radiative decay of the $\phi(1020)$ into $a_0(980)$ and $f_0(980)$ were proposed to disentangle compact (tetraquark) structures from hadronic molecules. Following Refs. [330, 331] the data from KLOE [332, 333], CMD-2 [334] and SND [335] seem to favor these mesons to be tetraquark states. This is also supported by a BESIII analysis of $J/\psi(1S) \to \gamma\pi^0\pi^0$ data [336] and by a measurement of $a_0(980) - f_0(980)$ mixing at BESIII [337]. In Ref. [38] these states are made of four-quark cores and virtual $K\bar{K}$ clouds at the periphery, a view that is challenged in Ref. [338] showing that radiative $\phi$ decay data are consistent with molecular structures of light scalars.

As mentioned already, the $f_0(1500)$ [40] and the $f_1(1285)$ [319] have been proposed as tetraquarks, the latter also as $K^*\overline{K}$ molecule [260], together with the $f_1(1420)$ [268].

Another potential candidate for a tetraquark is the $\phi(2170)$. As an excited $q\bar{q}$ state of the $\phi(1020)$, this meson should decay strongly into $K^+K^-$ and $K^0_SK^0_L$, and also $K^*\bar{K}^*$. However, these decays are not observed. Hence, based on its observed decay into $f_0(980)\phi(1020)$ this meson could be $ss\bar{s}\bar{s}$, see Ref. [339] where relative decay rates into various channels are predicted for both $q\bar{q}$ and tetraquark structures of the $f_0(980)$. However Refs. [340, 341] claim that the $\phi(2170)$ may not be a good candidate for the $ss\bar{s}\bar{s}$ state, but could qualify as an $su\bar{s}\bar{u}$ tetraquark [341].

More information on the $a_0(980)$, $f_0(980)$, $f_0(500)$ and $K^*_0(700)$ is provided by 'Scalar Mesons below 1 GeV' in the *Review*.

## 63.9 Baryonia

Nucleon-antinucleon ($N\bar{N}$) bound states and resonances (baryonium) were predicted a long time ago [342, 343], based on the strongly attractive $N\bar{N}$ meson exchange potential, which is obtained from the $NN$ one by multiplying with the $G$-parity of the exchanged meson. Several candidates had been reported in the seventies by experiments at CERN, BNL and KEK, some of them being indisputably statistically significant (for details on the model and on experiments, see [344, 345] and references therein). Most potential models predict a sequence of deeply bound isoscalar baryonia with quantum numbers $J^{PC} = 2^{++}$, $1^{--}$ and $0^{++}$, the latter being the mostly bound [346, 347]. The $f_2(1565)$ which is observed in $\bar{p}p$ annihilation only [348, 349], is a good candidate for the $2^{++}$ $\bar{p}p$ bound state.

Enhancements close to the $\bar{p}p$ threshold have also been reported in $B$ decays [350–352]. The strong signal in $J/\psi(1S) \to \gamma\bar{p}p$ [353–355] could be due to a $0^{-+}$ baryonium [356], but could also be generated by the $N\bar{N}$ final state interaction [357–360]. The strong energy dependence of the cross section in $e^+e^- \to \bar{p}p$ [361, 362] and $\bar{p}p \to e^+e^-$ [363] is attributed to the $N\bar{N}$ final state interaction [364] (see also our comments on the $\rho(1900)$ at the end of section 63.5.2). Alternative explanations to baryonia have been proposed for the signals in $B \to \bar{p}pK$, such as the dynamics of the fragmentation mechanism [351].

## 63.10 Conclusions

The ground states nonets $1^1S_0$, $1^3S_1$, $1^1P_1$, $1^3P_2$, $1^1D_2$ and $1^3D_3$ (see fig. 63.1) are complete and established (although more data are desirable for the $h_1(1415)$ in the $1^1P_1$ nonet). There are uncertainties and ambiguities in all other nonets . In the pseudoscalar sector the mere existence of the $\eta(1295)$ (possibly the radial excitation of the $\eta$) has been questioned. The $\eta(1440)$ consists of two states, one at 1405 MeV, the other at 1475 MeV, which could also be the manifestation of a single state decaying into two different final states. In this case the 1475 one could be the radial excitation of the $\eta'$ and the 1405 state the pseudoscalar ground state glueball predicted a long time ago by the bag model, although the lattice puts it above 2 GeV. In the $1^{++}$ nonet two isoscalars – the $f_1(1420)$ and $f_1(1510)$ – compete for the $s\bar{s}$ slot, the former perhaps being exotic, and the latter not being very well established. The first radial excitation $2^3S_1$ is complete, but questions have been raised as to the nature of the $\rho(1450)$ and its relation to the $\rho(1700)$. The classification of scalar mesons is not settled with the ones below 1 GeV being tetraquark states or part of the ground state nonet. The nature of the scalars in the 1.5 – 1.8 GeV region is unclear, obscured by the interference with the ground state scalar glueball predicted in this mass region by most theoretical models. For example, the $f_0(1500)$ and the $f_0(1710)$ could be $q\bar{q}$ states ($n\bar{n}$ and $s\bar{s}$, respectively) or made of gluons mixed with $q\bar{q}$. The $f_0(1500)$ or alternatively the $f_0(1710)$ have been proposed as mostly gluonic. Scalar mesons above the $f_0(1710)$ need experimental confirmation. The 1.4 - 1.6 GeV mass region is populated by one or two broad isovector states with exotic ($1^{-+}$) quantum numbers, the $\pi_1(1400)$ and $\pi_1(1600)$. The mass of latter is not far below theoretical predictions. Above 1.8 GeV mesons are broad and overlap, which complicates the data analyses. This makes the classification difficult, in particular the identification of the $2^{++}$ glueball.

## Acknowledgements

We thank Paolo Gauzzi, Bertram Kopf, Mikhail Mikhasenko and Anton Rebhan for carefully reading the manuscript, and for their detailed and constructive suggestions. We have also benefitted from several reviews published earlier in the *Review of Particle Physics*, jointly with Thomas Gutsche, Christoph Hanhart and Stefan Spanier, which have been merged into the current article.

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