84. Pentaquarks

Revised September 2021 by M. Karliner (Tel Aviv U.) and T. Skwarnicki (Syracuse U.).

Experimental searches for pentaquark hadrons comprised of light flavors have a long and vivid history. No undisputed candidates had been found in 50 years. The first wave of claimed observations of pentaquark candidates containing a strange antiquark occurred in the early seventies, see e.g. a review in the 1976 edition of Particle Data Group listings for $Z_0(1780)$, $Z_0(1865)$ and $Z_1(1900)$ [1]. The last mention of these candidates can be found in the 1992 edition [2] with the perhaps prophetic comment "the results permit no definite conclusion - the same story for 20 years. [...] The skepticism about baryons not made of three quarks, and lack of any experimental activity in this area, make it likely that another 20 years will pass before the issue is decided." A decade later, a second wave of observations occurred, possibly motivated by specific theoretical predictions for their existence [3–5]. The evidence for pentaquarks was based on observations of peaks in the invariant mass distributions of their decay products. More data and subsequent more sensitive experiments did not confirm these claims [6]. In the last mention of the best known candidate from that period, $\Theta(1540)^+$, the 2006 Particle Data Group listing [7] included a statement: "The conclusion that pentaquarks in general, and that Θ^+, in particular, do not exist, appears compelling." which well reflected the prevailing mood in the particle physics community until a study of $\Lambda_b^0 \to J/\psi pK^-$ ($J/\psi \to \mu^+\mu^-$) decays by LHCb [8] (charge conjugate modes are implied). From an analysis of 3 fb^{-1} Run 1 data at 7 and 8 TeV at the LHC, the LHCb collaboration reported a significant $J/\psi p$ structure in $\Lambda_b^0 \to J/\psi pK^-$ decays [8]. The exotic character of this structure, with the minimal quark content of $uudc\bar{c}$, was demonstrated in a nearly model-independent way in Ref. [9], where it was shown that the $J/\psi p$ mass ($m_{J/\psi p}$) peak near 4450 MeV was too narrow to be accounted for by $\Lambda^* \to pK^-$ reflections (Λ^* denotes a generic Λ excitation), reinforcing the results from the earlier model-dependent six-dimensional amplitude analysis of invariant masses and decay angles describing the Λ_b^0 decay in the same data [8]. Even though not apparent from the $m_{J/\psi p}$ distribution, the amplitude analysis also required a second broad $J/\psi p$ state to obtain a good description of the data, which peaked at $4380 \pm 8 \pm 29$ MeV with a width of $205 \pm 18 \pm 86$ MeV and a fit fraction of $(8.4 \pm 0.7 \pm 4.2)\%$.

The LHCb 6 fb^{-1} Run 2 LHC data at 13 TeV, together with the improvements in the data selection for both runs, resulted in a nine-fold increase in the number of reconstructed $\Lambda_b^0 \to J/\psi pK^-$ decays (246,000 events) [10]. When fit with the same six-dimensional amplitude model, the enlarged data sample gives consistent results for the $P_c(4450)^+$ and $P_c(4380)^+$ parameters, corroborating the compatibility of the data samples. However, the two-state interpretation of the data is contradicted by the observation of new narrow $J/\psi p$ structures which are too faint to have been significant in the Run 1 data analysis. A second horizontal band is observed in the Dalitz plot (Fig. 84.1) near 4312 MeV in the $J/\psi p$ mass.

The 4450 MeV structure also appears to consist of two narrower peaks at 4440 and 4457 MeV. Performing a rigorous six-dimensional amplitude analysis of these faint $J/\psi p$ structures is challenging and has not been accomplished yet. Fortunately, the newly observed peaks are so narrow that it is not necessary to construct an amplitude model to prove that these states are not artifacts of interfering Λ^* resonances, as was previously demonstrated in Ref. [9]. Their masses and widths have been characterized by the LHCb (see Table 84.1) from one-dimensional fits to $J/\psi p$ mass distributions, with different levels of suppression of the Λ^* contributions, which peak at the lower pK^- masses (Fig. 84.1). Such analysis is not sensitive to any broad $J/\psi p$ contributions like $P_c(4380)^+$. The histograms analyzed by the LHCb are available in tabular form at

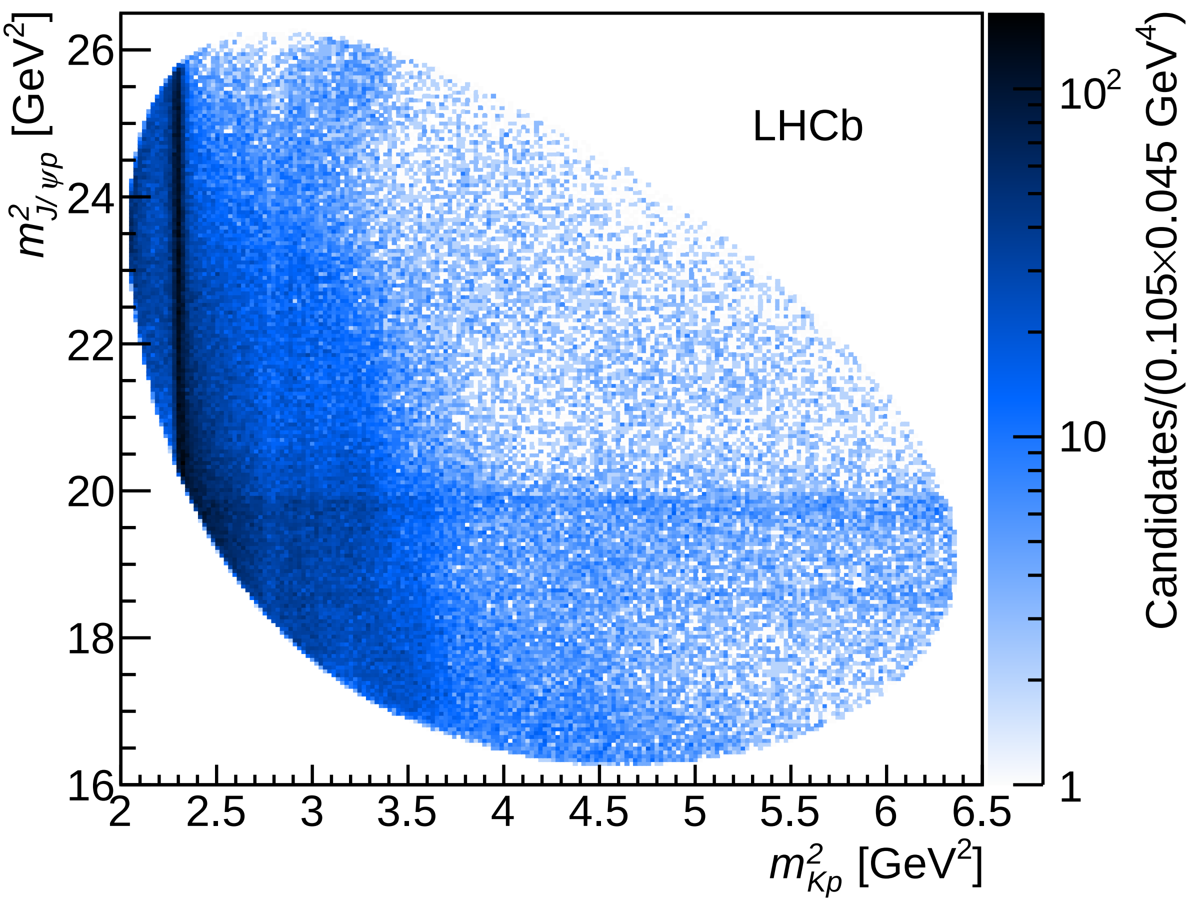

Figure 84.1: Dalitz plot distributions for $\Lambda_b^0 \to J/\psi\, pK^-$ decays as observed by LHCb.

https://www.hepdata.net/record/89271.

The fit chosen by the LHCb for the central mass and width values is displayed in Fig. 84.2. The $P_c(4312)^+$ state peaks right below the $\Sigma_c^+\overline{D}^0$ threshold and has statistical significance over 7.6σ. The $P_c(4457)^+$ state peaks right below the $\Sigma_c^+\overline{D}^{*0}$ threshold, while the $P_c(4440)^+$ state peaks about 20 MeV below it. The significance of the two-peak versus one-peak hypothesis for the 4450 MeV structure is over 5.4σ, rendering the single peak interpretation of this region obsolete. The six-dimensional amplitude analysis reported in Ref. [8], which provided evidence for the $P_c(4380)^+$ state, is obsolete since it used the single $P_c(4450)^+$ state and it lacked the $P_c(4312)^+$ state. Furthermore, it used the helicity formalism in which the half-integer spin of the proton was not aligned properly between the different Λ_b decay chains [11, 12]. The newer one-dimensional analysis by LHCb [10] was not sensitive to wide P_c^+ states. The LHCb result from the six-dimensional amplitude analysis of the Cabibbo suppressed channel $\Lambda_b^0 \to J/\psi\, p\pi^-$ [13], which contains a statistically marginal evidence for the sum of the P_c^+ and the $Z_c(4200)^-$ contributions, took extensive input from Ref. [8] and, like the $P_c(4380)^+$ state, should be treated with caution until the both amplitude analyses are completed on the enlarged data sets with the modified helicity formalism.

While $\Sigma_c\bar{D}^{(*)}$ states had been predicted [14–17] before the first LHCb results [8], after these results became known, many theoretical groups interpreted the $P_c(4450)^+$ and $P_c(4380)^+$ states in terms of diquarks and triquarks as building blocks of a compact pentaquark [18–24]. In a different strategy, a tentative attempt has been made to treat the full 5-body dynamics, leading to states below the lowest threshold for spontaneous dissociation [25]. In the first implementation

Table 84.1: Summary of the narrow P_c^+ properties, interpreted as Breit-Wigner resonances. The central values are based on the fit displayed in Fig. 84.2.

State	M [MeV]	Γ [MeV] (95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$ (< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$ (< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$ (< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$

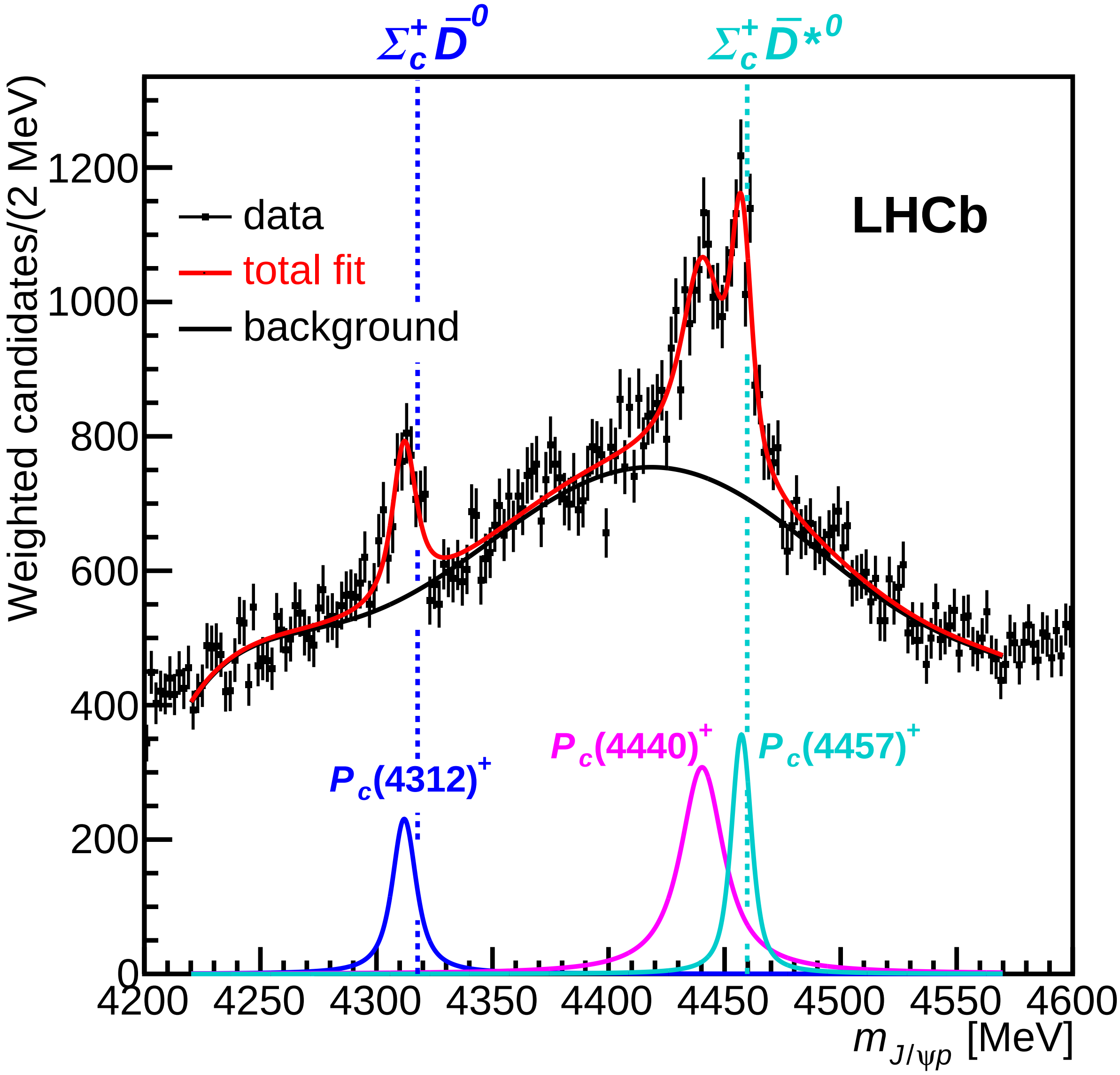

Figure 84.2: Fit to the $J/\psi p$ mass distribution, in which events were weighted to suppress $\Lambda^* \to pK^-$ backgrounds, of three Breit-Wigner functions and a sixth-order polynomial background. This fit was used to determine the central values of the masses and widths of the P_c^+ states reported by LHCb. The mass thresholds for the $\Sigma_c^+\overline{D}^0$ and $\Sigma_c^+\overline{D}^{*0}$ final states are superimposed.

of the former approach [18], the pentaquark mass splitting was generated mostly by the change of angular momentum between the sub-components (L) from zero to one, which would also make the heavier state narrower and of opposite parity. Explicit modeling of multiquark systems [26] questions if centrifugal barrier factor provides enough width suppression via spatial separation of c and $\bar{c}$ quarks at these masses, as the phase space for $J/\psi p$ decay is very large (more than 400 MeV). Also, the observed mass splitting was too small to be only due to the mechanism proposed in Ref. [18] and required fine-tuning of such models. A variation of this model, in which the heavy (cu) diquark couples with heavy $\bar{c}$ to form colored triquark attracting the light diquark (ud), has been re-implemented for the narrow P_c^+ states [27]. In this model, the $P_c(4440)^+$ and $P_c(4457)^+$ states are accommodated via spin-orbit interactions for the $L = 1$ states, while the $P_c(4312)^+$ is one of the $L = 0$ states. However, the mass prediction for the latter is off by (-72 ± 29) MeV [27]. This work was later extended to $SU(3)_F$ [28]. The width dilemma becomes more severe in view of the narrow widths of the newly observed states (Table 84.1), especially for the $L = 0$ $P_c(4312)^+$ state, and requires a different origin of potential barrier between c and $\bar{c}$ than angular momentum [27,29], which remains a subject of theoretical controversy. Measurement of spin-parity of $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$ will be crucial for testing these and alternative theoretical ideas discussed in the following.

More effective width suppression mechanism is offered by a loosely bound charmed baryon-anticharmed meson molecular model, in which c and $\bar{c}$ can be separated by much larger distances, resulting in a smaller probability of them getting close enough to each other in order to make a J/ψ. Since molecular binding energy cannot be large, such molecules are in S-wave, so their masses must be near the sum of the baryon and meson masses and their spin and parity are inherited from their constituent hadrons [17]. The mass coincidence of the $P_c(4312)^+$ and of $P_c(4457)^+$ states, with the two related thresholds, $\Sigma_c^+ \overline{D}^0$ and $\Sigma_c^+ \overline{D}^{*0}$, provides very strong experimental evidence in favor of this interpretation. Given how close $P_c(4312)^+$ is to the $\Sigma_c^+ \overline{D}^0$ threshold, it might be a virtual rather than a bound state [30]. Since the spins of Σ_c^+ and of $\overline{D}^{*0}$ can be combined in two different ways, the narrow $P_c(4440)^+$ peak also finds natural explanation in this physical picture. It cannot be a virtual state since it is sufficiently below the $\Sigma_c^+ \overline{D}^{*0}$ threshold.

It is worth stressing that other baryon-meson combinations, $\Lambda_c^{(*)+} \bar{D}^{(*)0}$ and $\chi_{cJ} p$ are not expected to bind [14, 31]. Before the first pentaquark observation [8] heavy quark symmetry was used to show that, in addition to the three $\Sigma_c \bar{D}^{(*)0}$ states, one expects four $\Sigma_c^* \bar{D}^{(*)0}$ states, for a total of seven [32]. Indeed, additional states at, or below, the $\Sigma_c^{*+} \overline{D}$ and $\Sigma_c^{*+} \overline{D}^*$ thresholds, are expected [33–36]. Since Σ_c^{*+} width is likely around 15 MeV [37]. more than the width of either $P_c(4312)^+$ or $P_c(4457)^+$, it is important to keep in mind that a molecule is typically as broad as the constituents[1] [38–40]. For a review on hadronic molecules, see [41, 42].

It is useful to consider the $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$ narrow pentaquarks together with several analogous exotic states with hidden charm and bottom in the meson sector. This provides additional significant motivation for the molecular model. At least five exotic mesons are close to thresholds of two heavy-light mesons: $X(3872)$ [43–46], $Z_b(10610)$ and $Z_b(10650)$ in the bottomonium sector [47–51] and $Z_c(3900)$ [52–56] and $Z_c(4020)$ [57–59] in the charmonium sector (see Table II in Ref. [60]; for reviews of experimental information see Ref. [61,62], as well as *Spectroscopy of Mesons Containing Two Heavy Quarks* and *Heavy Non-$q\bar{q}$ Mesons* in the current Review of Particle Properties. These states share several important features: a) their masses are near thresholds and their spin and parity correspond to S-wave combination of the two mesons; b) they are very narrow, despite very large phase space for decay into quarkonium + pion(s); c) the branching fractions for “fall apart" mode into two mesons are much larger than branching fractions

[1] This feature gets changed if systems are more deeply bound.

for decay into quarkonium and pion(s). So far, there is no experimental evidence for states at two pseudoscalar thresholds ($D\bar{D}$ and $B\bar{B}$), implying that pseudoscalar exchange is essential for binding in meson-meson systems.

The above provide a strong hint that these states are deuteron-like loosely bound states of two heavy mesons [63–71]. It is then natural to conjecture that similar bound states might exist of two heavy baryons [72,73], or a meson and a baryon or a baryon and an antibaryon, leading to a rather accurate prediction of the $P_c(4457)^+$ mass as $3/2^-$ $\Sigma_c\bar{D}^*$ molecule (the mass threshold is 4460 MeV for $\Sigma_c^+ D^{*0}$ and 4464 MeV for $\Sigma_c^{++} D^{*-}$) [17,60], following similar predictions obtained in a wider framework of doubly heavy baryon-meson hadronic molecules, which might include mixtures of various two-hadron states [14–16,31]. However, single pion exchange is not possible in $\Sigma_c^+\overline{D}^0$ system, thus the existence of $P_c(4312)^+$ points to importance of vector or two-pion exchanges in baryon-meson molecules. Two-pion exchange in $D\bar{D}$ system is highly suppressed, because the intermediate state is $D^*\bar{D}^*$, which is 282 MeV heavier than $D\bar{D}$. On the other hand, there is little suppression in the $\Sigma_c\bar{D}$ system, because the dominant intermediate state is $\Lambda_c\bar{D}^*$ which is just 25 MeV lighter than $\Sigma_c\bar{D}$ [74]. In a generic hadronic molecule it is essential that the two hadrons are heavy, in order to minimize the repulsive kinetic energy [72,73,75].

Following the initial LHCb discovery [8], several groups carried out a detailed analysis of the P_c^+ states as hadronic molecules [76–85] followed by further analyses [35,86–110] after the updated LHCb results [10]. Partial widths of all the allowed decay channels for the P_c states have been estimated within a specific model in the molecular picture [111]. The most striking result is that $P_c(4312)$ decays are totally dominated by the $\bar{D}^*\Lambda_c$ channel. This channel is also expected to be very prominent in decays of $P_c(4440)$ and $P_c(4457)$. A recent theoretical analysis reaches an opposite conclusion, on the basis of the data available so far, expecting a rather small signal in the $\bar{D}^{(*)}\Lambda_c$ channels [112]. Obviously, experimental determination of the branching fraction of these channels is of high priority.

The P_c states have also been-interpreted as so called hadro-charmonium [113], a bound state of relatively compact charmonium states with light hadronic matter. It was proposed that $P_c(4440)^+$ and $P_c(4457)^+$ are spin-split $\psi(2S)p$ bound states with $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$, while $P_c(4312)^+$ is a $\chi_{c0}p$ bound state with $J^P = \frac{1}{2}^+$ [114]. While very interesting from the theoretical point of view, it is not at all clear why the binding energies between charmonia and the nucleon should conspire to produce states so close to the $\Sigma_c\bar{D}$ and $\Sigma_c\bar{D}^*$ thresholds. Moreover, the predicted widths of $P_c(4440)^+$ and $P_c(4457)^+$ are too big by a factor $\sim$ 2-3. One should also keep in mind that the molecular and hadro-charmonium pictures provide opposite predictions for the parity of $P_c(4312)^+$. In principle LHCb can check the spin and parity through partial wave analysis, but at present it is not known if systematic uncertainties can be sufficiently reduced to make such an analysis conclusive.

Shortly after the initial experimental discovery it was conjectured that the $P_c(4450)^+$ reflects the presence of a triangle singularity near the $\chi_{c1}p$ threshold [115–118]. These explanations are no longer viable, since the $P_c(4440)^+$ mass is not at any threshold and the $P_c(4312)^+$ and $P_c(4457)^+$ peak slightly below the $\Sigma_c^+\overline{D}^0$ and $\Sigma_c^+\overline{D}^{*0}$ thresholds. The $P_c(4457)^+$ mass is exactly at $\Lambda_c^{*+}\overline{D}^0$ threshold, but LHCb has demonstrated that the observed peaking is narrower in the data than expected from the triangle-diagram when a realistic width of the excited D_s^- state exchanged in the triangle is used (Supplemental Material in Ref. [10]).

More extensive pre-2019 reviews of some of the theoretical issues can be found in Refs. [119,120]. Two recent relevant reviews are Refs. [121,122].

So far the P_c^+ states have been observed by only one experiment in only one channel. It is essential to explore other possible experimental channels, such as $P_c \to \Lambda_c\bar{D}^{(*)}$, $\eta_c p$ [112]. These channels are however much more experimentally challenging than $P_c \to J/\psi p$. Proposals have also been made to search for heavy pentaquarks in photo-production [123–130]. Ref. [131] discusses photoproduction within the string-junction physical picture of the pentaquarks. Photoproduction is also related to recent work on $J/\psi(\eta_c)N$ scattering on the lattice [132] and on computation of $J/\psi(\eta_c)N$ and $\Upsilon(\eta_b)N$ cross sections [133]. In addition, pentaquark production has been discussed in the context of antiproton-deuterium collisions [134], of heavy ion collisions at LHC [135], in pA collisions [136] and in pion-induced processes [137–139]. The GlueX Collaboration reported negative search results for the P_c^+ states in photo-production at JLAB [140]. Within the large experimental errors and considerable theoretical model dependence these results do not contradict the molecular interpretations of the narrow P_c^+ states, especially in view of the P_c decay being likely dominated by $\Lambda_c\bar{D}^{(*)}$ modes [111, 141]. It has been suggested to determine the pentaquark photo-couplings and branching ratios by measuring the polarization transfer between the incident photon and the outgoing proton in the exclusive photo-production of J/ψ near threshold [142].

As noted in e.g. [17], bottom analogues of the P_c^+ might well exist, but experimental search for such states involves very significant challenges. It is therefore hardly surprising that they have not been observed so far. A detailed discussion is beyond the scope of the current review.

Recently, the LHCb collaboration has obtained a 3.1σ evidence for a $P_c^+ \to J/\psi\, p$ state in the four-dimensional amplitude analysis of about 800 $B_s^0 \to J/\psi\, p\bar{p}$ ($J/\psi \to \mu^+\mu^-$) decays [143]. The mass of the state, $4337^{+7}_{-4} \pm 2$ MeV, is not compatible with the the $P_c(4312)^+$ state at 3.1 standard deviations. The width is relatively small, $29^{+26}_{-12} \pm 14$ MeV. The present data do not provide sufficient discrimination between various J^P assignments. Its mass is about 19 MeV higher than the $\Sigma_c^+\bar{D}^0$ threshold and about 16 MeV lower than the $\chi_{c0}\,p$ threshold. At present, no specific explanation for this structure has been proposed, but some intriguing ideas have been raised in Ref. [144]. Since the statistical significance of this evidence is marginal, more data are required before this state is considered experimentally established.

Similarly inconclusive 3.1σ evidence for a $P_{cs}^0 \to J/\psi\,\Lambda$ structure was observed by the LHCb collaboration in the six-dimensional amplitude analysis of about 1750 $\Xi_b^- \to J/\psi\,\Lambda K^-$ ($J/\psi \to \mu^+\mu^-$, $\Lambda \to p\pi^-$ decays [12]. The strange counterparts of P_c^+ states have been predicted in both molecular and compact pentaquark models [15, 98, 100, 145–147]. When interpreted as a single peak, its mass, $4458.8 \pm 2.9^{+4.7}_{-1.1}$ MeV, is 15 MeV above the $\Xi_c^{0\prime}\bar{D}^0$ threshold and 18 MeV below the $\Xi_c^0\bar{D}^{*0}$ threshold. Its width, $17.3 \pm 6.5^{+8.0}_{-5.7}$ MeV, is relatively narrow, thus plausibly could be interpreted as a loosely bound $\Xi_c^0\bar{D}^{*0}$ state. However, in the latter model two states are expected with J^P equal to $1/2^-$ and $3/2^-$. In fact, the LHCb data are consistent with such hypothesis, under which 4454.9 ± 2.7 MeV ($\Gamma = 7.5 \pm 9.7$ MeV) and 4467.8 ± 3.7 MeV ($\Gamma = 5.2 \pm 5.3$ MeV) mass (width) estimates are obtained (statistical errors only). It is worth noting, that the $SU(3)$ flavor structure of $\Xi_c^0\bar{D}^{*0}$ states is different from that of $\Sigma_c^+\bar{D}^{*0}$ states, since Ξ_c^0 belongs to the charmed baryon triplet, unlike Σ_c^+, which together with $\Xi_c^{0\prime}$, belong to the charmed baryon sextet. More data are required to experimentally establish this structure, clarify its composition, and determine the related quantum numbers.

If $P_{cs}(4459)^0$ indeed is a $\Xi_c^0\bar{D}^{*0}$ molecule, there likely exists another state, corresponding to $\Xi_c'\bar{D}^*$ [15, 98, 100, 145–147], with a mass shifted upwards by approximately $\Xi_c' - \Xi_c$ mass difference [148], i.e. about 108 MeV [37]. Experimental determination whether yet additional $\Xi_c^{(*,\prime)}\bar{D}^{(*)0}$ states actually exist will provide a useful testing ground for the various theoretical approaches which have been used to predict them.

In hidden-charm exotics discussed above, the charmed and the anticharmed quarks can form a charmonium and thereby decouple from the light quarks. The molecular model provides an efficient mechanism to suppress such decays, thus making these states narrow. This type of exotic states, should not be confused with exotic hadrons containing two heavy quarks, rather than a heavy quark and a heavy antiquark. Such a decoupling is impossible in exotics which contain two heavy quarks. This has far-reaching consequences.

The first exotic hadron containing two heavy quarks has recently been reported by LHCb [149, 150] and deserves a separate review. Here we briefly discuss only the essential distinction vs. hidden-charm pentaquarks. The reported T_{cc}^{+} state has quark content $cc\bar{u}\bar{d}$, $I = 0$ and likely $J^P = 1^+$, and mass about 300 keV below the D^0D^{*+} threshold. Its width is a fraction of an MeV. The phase space available for decay is tiny due to absence of the decoupling mechanism. Therefore the narrow width of T_{cc}^{+} is natural in both the compact tetraquark model and in those molecular models which predict the mass very close to the D^0D^{*+} threshold. On the other hand, for sufficiently heavy quarks, e.g. $bb\bar{u}\bar{d}$, such open heavy-flavor exotics are genuine compact tetraquarks which are *stable* under strong interactions; see discussion and references in Refs. [149, 150].

It is quite possible that compact tetraquark states exist also for states with hidden-flavor, but such states are expected to be broader because of the decoupling mechanism. In fact, there is plenty of evidence for them in form of mass structures in $J/\psi\,\phi$ [151–156], $J/\psi\,J/\psi$ [157], $Z_c(4430)^+ \to \psi(2S)\pi^+$ [158–160] and others, although alternative explanations of these structures have been also proposed. Analogous configurations may also exist for pentaquarks, but there is no firm evidence for them yet.

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